Quiz 22 Extending Bases.

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We are given that a vector space V over a field K has a basis $H = \begin{pmatrix} u_1 & u_2 & u_3 & u_4 \end{pmatrix}$. Answer the following questions.

1. Let $w_1 = u_1 + u_2 + u_3$ and $w_2 = u_2 + u_3 + u_4$. Choose two vectors w_3 and w_4 such that $B = \begin{pmatrix} w_1 & w_2 & w_3 & w_4 \end{pmatrix}$ is also a basis of V. Suggestion: Try to find the simplest vectors which will span V. Then use a known theorem.

   Answer: Take $w_3 = u_2$ and $w_4 = u_3$. Then $u_1 = w_1 - w_3 - w_4$ and $u_4 = w_2 - w_3 - w_4$. Hence they span V and since dimension of V is 4, they are a basis.

2. Write $H = BP$ and hence deduce $[H]_B = \begin{pmatrix} [u_1]_B & [u_2]_B & [u_3]_B & [u_4]_B \end{pmatrix}$.

   Answer: The answers depend on the choice of w_2, w_3. The coordinate vectors are shown above for my choice.

3. Also find $[B]_H$. Hint: This is easier to find and useful for the above question.

   Answer: The expressions of the w-vectors gives $B = HQ$ for some matrix Q. The columns of Q are the desired coordinate vectors. The above P is simply Q^{-1}.

4. For meditation: Think about these statements: If $B = \begin{pmatrix} v_1 & v_2 & \cdots v_n \end{pmatrix}$ is a basis of a vector space V, then the vectors $(v_1, v_2, \cdots, v_n, v)$ are necessarily linearly dependent for every $v \in V$. Moreover we can drop exactly one of v_i from $i = 1, \cdots, n$ such that the resulting sequence is a new basis for V. The one vector being dropped may not be unique.

   Answer: This is called the exchange principle.