Quiz 23 Eigenvectors and eigenvalues.

Ma322 Fall 2018 Avinash Sathaye

Consider a matrix $A = \begin{pmatrix} 3 & 5 \\ 4 & -5 \end{pmatrix}$.

Answer the following questions.

1. Determine the characteristic polynomial $P(t) = \det(A - tI)$ for A.

   Answer: Use the simple formula for a 2×2 matrix, namely $P(t) = t^2 - (3-5)t + ((3)(-5) - (5)(4)) = t^2 + 2t - 35$.

2. Determine all the eigenvalues of A, i.e. determine all the roots of $P(t)$.

   Answer: $P(t) = (t+7)(t-5)$ so the roots are $t = -7, t = 5$.

3. For each eigenvalue $t = s$, find an eigenvector v_s such that $v_s \neq 0$ and $Av_s = sv_s$.

   Answer: For $t = -7$ find a basis for $Nul(A + 7I) = Nul(\begin{pmatrix} 10 & 5 \\ 4 & 2 \end{pmatrix}$ and an easily guessed answer is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$. Similarly, for the value $t = 5$, we get an answer: $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

4. For meditation: If $P(t)$ is the characteristic polynomial of a matrix A, the the famous Cayley Hamilton theorem says that $P(A) = 0$. Test it for the above matrix and others square matrices.

   Answer: What could be a proof?