Quiz 24 Eigenvectors and eigenvalues II.

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Let $P = \begin{pmatrix} v_1 & v_2 \end{pmatrix}$ be a 2×2 matrix whose columns are $v_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.
Also let A denote a 2×2 matrix.

Answer the following questions.

1. Write down the condition that v_1 is an eigenvector of A belonging to the eigenvalue $1/3$.

   Answer: $Av_1 = \frac{1}{3}v_1$ or $A\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

2. Write down the condition that v_2 is an eigenvector of A belonging to the eigenvalue $-1/2$.

   Answer: $Av_2 = -\frac{1}{2}v_2$ or $A\begin{pmatrix} 3 \\ 4 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

3. Using the conditions, determine a diagonal matrix D such that $AP = PD$. Deduce the formula for A.

   Answer: $D = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$ works. Then $A = PDP^{-1}$.

4. For meditation: By multiplying the matrices of the formula, it is possible to get the explicit matrix A. When we consider the linear transformation $L(X) = AX$ then we can effectively calculate the sequence of vectors $w, Aw, A^2w, \cdots, A^nw, \cdots$. It is often important to decide if this sequence has a limiting vector.

   Answer: Try it on the A above.