

Quiz 25 Using Eigenvectors.

Ma322 Fall 2018 Avinash Sathaye

$$\text{Let } A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 5 \end{pmatrix}$$

Answer the following questions.

1. Determine all the eigenvalues of A . **No calculations are needed.**

Answer: The characteristic polynomial shall be clearly $-(\lambda - 1)^2(\lambda - 5)$, so the values are 1, 5.

2. For each of the eigenvalues, determine the dimension of the corresponding eigenspaces.

Answer: Both are 1-dimensional, since the corresponding ranks of $A - \lambda I$ are easily seen to be 2 each.

3. Does the above calculation show that A is diagonalizable? Why?

Answer: Sum of the eigenspace dimensions is only 2 which is less than 3 the dimension of the column space of A . So, no!

4. **For meditation:** What will happen if we change the entries 2, 3, 4 into zeros? What can we say if an $n \times n$ matrix has n distinct eigenvalues? Is it diagonalizable or it may fail to be diagonalizable? What are the possible values for the sum of dimensions of eigenspaces of an $n \times n$ matrix?

Answer: Try various examples of triangular matrices as above and see what happens.