

Quiz 26 Complex Eigenvalues.

Ma322 Fall 2018 Avinash Sathaye

Let $A = \begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix}$

Answer the following questions.

1. Determine the eigenvalues of A , **They are not real!**

Answer: The characteristic polynomial is $\lambda^2 + \lambda + 1$, so the e-values are $\frac{-1 \pm \sqrt{3}}{2}$.

2. Choose one of the e-values, say $p + iq$ and determine a corresponding e-vector written as $v_1 + iv_2$. Explicitly write down your choice.

Answer: e-value is $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and the corresponding e-vector is $\begin{pmatrix} 3 \\ -\frac{3}{2} \end{pmatrix} + i \begin{pmatrix} 0 \\ \frac{\sqrt{3}}{2} \end{pmatrix}$.

3. Now we can create the standard form M from the matrix $A = PMP^{-1}$ where P is the matrix with columns v_1, v_2 respectively and M is assembled from $p + iq$ by a standard formula.

Write these out.

Answer: $P = \begin{bmatrix} 3 & 0 \\ -3/2 & -1/2\sqrt{3} \end{bmatrix}$ and $M = \begin{bmatrix} -1/2 & -1/2\sqrt{3} \\ 1/2\sqrt{3} & -1/2 \end{bmatrix}$.

4. **For meditation:** Canonical Forms of linear transformations. The above work is a sample of how to classify infinitely many possible matrices of linear transformations into a finite number of possible types and then produce a corresponding canonical form for each type. For transformations on two dimensional real spaces, there are only three types, two real roots, two cases of a real double root and complex conjugate roots. Study what happens in higher dimensions.

Answer: Google or ask!