Quiz 27 Difference Equations

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Let $v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and let $P = \begin{pmatrix} v_1 & v_2 \end{pmatrix}$.

Further assume that A is a 2×2 matrix with v_1, v_2 as e-vectors with e-values $1, 0.3$ respectively.

Answer the following questions.

1. Briefly explain why A must be equal to PDP^{-1} where $D = \begin{pmatrix} 1 & 0 \\ 0 & 0.3 \end{pmatrix}$.

   Answer: By definition $AP = PD$. Result follows after right multiplication by P^{-1}.

2. Write $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = c_1 v_1 + c_2 v_2$. In other words $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = [e_1]_B$ where $B = [v_1 \;\; v_2]$. Deduce the formula for $A^n e_1$ as an explicit expression in n.

   Answer: The first answer is the first column of P^{-1}, namely $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

   Then we get: $A^n e_1 = c_1(1)^n v_1 + c_2(0.3)^n v_2 = 3 \begin{pmatrix} 2 \\ 1 \end{pmatrix} + 2(0.3)^n \begin{pmatrix} 5 \\ 3 \end{pmatrix}$.

3. Determine the limit: $\lim_{n \to \infty} A^n e_1$.

Answer: The answer is $3v_1 = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$ since the second part goes to zero.

4. For meditation: In general, an $m \times m$ matrix A gives a linear transformation from $\Re^m$ to $\Re^m$. It can be investigated by starting with a vector w_0 and iteratively calculating $w_n = A^n w_0$. Such sequences are called dynamical systems. The sequence (w_n)can be effectively understood if w_0 can be expressed as a combination of e-vectors. In particular the resulting limit, if any, has significant applications in dynamical systems. These systems appear in diverse fields including Biology, Computer Science, Engineering, Social Sciences and so on.

   Answer: Google or ask!