Quiz 28 Orthogonal Bases.

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Let v_1, v_2, v_3 be the columns of the matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & -4 & 1 \\ 1 & 0 & -10 \end{bmatrix}$.

Answer the following questions.

1. Calculate the I.P. matrix $A^T A$ and briefly explain why the three vectors are mutually orthogonal. Why is $B = (v_1 \ \ v_2 \ \ v_3)$ a basis of $\Re^3$?

   Answer: We get $A^T A = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 105 \end{bmatrix}$. Since this is a diagonal matrix, our vectors are orthogonal. They form a basis since they are orthogonal and non zero.

2. Use above to determine the coordinate vector $[v]_B$ where $v = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

   Answer: The three coordinates of v are in order $7/21, -2/20, 7/105$. The easiest way to calculate is to evaluate $v^T A = \begin{pmatrix} 7 & -2 & 7 \end{pmatrix}$ and divide by the diagonal entries of the I.P. matrix above.

3. For meditation: Orthogonal bases are very nice, since it is easy to determine coordinate vectors in such bases. So, a natural question is how to convert a given basis (or even a spanning set) into an orthogonal set of vectors by simple row operations. The process is called the GramSchmidt algorithm, but we present a much simpler version not in the book!