Quiz 29 Inner Products.

Ma322 Fall 2018 Avinash Sathaye

Let V be a vector space with basis $B = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$.
Also suppose that V has the following inner product matrix relative to B:

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 3 \\ 0 & 3 & 11 \end{bmatrix}.$$

Answer the following questions.

1. Determine the inner product $< v_1, v_2 + av_1 >$ where a is a parameter. Then choose a value of a for which $v_2 + av_1$ is perpendicular to v_1.

   Set $w_2 = v_2 + av_1$ using your value of a.

   Answer: The inner product is $2 + a(1)$ and hence $a = -2$.

2. Determine the inner product $< w_2, v_3 + bw_2 >$ where b is a parameter. Then determine a value of b for which $v_3 + bw_2$ is perpendicular to w_2.

   Set $w_3 = v_3 + bw_2$ using your value of b.

   Answer: The inner product is $3 + b(1)$ and hence $b = -3$.

3. For meditation: It can be verified that v_1, w_2, w_3 form an orthogonal set of vectors. They are almost orthonormal, except that w_3 is not a unit vector.

   This is the main idea of the GramSchmidt algorithm. We present a process similar to Gauss-elimination to streamline the work.

   Answer: See notes and learn in class.