Quiz 30 Easier Gram-Schmidt.

Ma322 Fall 2018 Avinash Sathaye

Let V be a vector space with basis $B = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$.
Also suppose that V has the following inner product matrix relative to B:

$$< B, B >= M = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 10 & 5 \\ 2 & 5 & 6 \end{bmatrix}.$$

Answer the following questions.

1. Carry out the standard Gauss elimination on the augmented matrix $(M|I)$.

   At the end, report the matrix that comes from I, i.e. the matrix of the last three columns.

   Answer: We get $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -5 & 1 & 1 \end{bmatrix}$

2. Let R be the transpose of the above answer. Calculate the product BR. Your answer will be three linear combinations of v_1, v_2, v_3.

   Answer: We get $\begin{pmatrix} v_1 & -3\,v_1 + v_2 & -5\,v_1 + v_2 + v_3 \end{pmatrix}$.

3. For meditation: This gives an easy way to convert any basis of a vector space into an orthogonal basis. Indeed this can be done on any sequence of vectors, except the resulting sequence may have one or more zero vectors and the non zero vectors form a basis of the span of the original sequence.

   Thus, this is an alternate effective way to find a basis and/or null space.

   Answer: See notes and learn in class.