Quiz 31 General Inner Products.

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Let P_2 be the vector space with basis $B = \left(1, t, t^2 \right)$.

Suppose that an inner product of two polynomials $p(t), q(t)$ is defined by $< p, q >= p(-1)q(-1) + p(0)q(0) + p(1)q(1)$.

Thus, for example: $< 1 + t, 1 + t^2 >= 0 \cdot 2 + 1 \cdot 1 + 2 \cdot 2 = 0 + 1 + 4 = 5.$

Answer the following questions.

- Determine the I.P. matrix M for the basis B using the above defined inner product.

  Answer: We get $\begin{bmatrix} 3 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$

- Carry out the Gram-Schmidt process by the easy method of determining the REF for $(M|I)$.

  Report the upper triangular matrix R from the process.

  Answer: $\begin{bmatrix} 1 & 0 & -2/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

- Using R give the orthogonal basis of P_2 obtained from B by the above process.

  Hint: Remember that you calculate BR.

Answer: $[1, t, -2/3 + t^2]$.

- For meditation: Different inner products on the same vector space produce different orthogonal bases. These give a way to measure how close the two polynomials (or functions) are, by checking the norm of the difference of their coordinate vectors in the orthogonal basis. The above inner product estimates how close two functions are at the three chosen points $-1, 0, 1$.

  Answer: See notes and learn in class.