Quiz 6 Linear Dependence.

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Suppose that you are given an augmented matrix and its RREF:

$$M = \begin{bmatrix} 1 & 4 & 0 & 5 & 1 \\ -4 & 5 & -2 & 1 & -6 \\ 0 & -1 & 5 & -1 & 5 \\ -5 & 2 & -1 & -3 & -6 \end{bmatrix} \Rightarrow \quad M^* = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Answer the following questions.

- Define what is meant by three vectors v_1, v_2, v_3 to be linearly dependent. **Answer:** The given vectors v_1, v_2, v_3 are linearly dependent if there are scalars(constants) c_1, c_2, c_3 such that $c_1v_1 + c_2v_2 + c_3v_3 = 0$ and at least one of c_1, c_2, c_3 is non zero.

  An alternate formulation is that one of the three vectors is a linear combination of the other two.

- Let $S = \{v_1, v_2, v_3, v_4, v_5\}$ be the columns of M listed in order. Prove that v_1, v_2, v_4 are linearly dependent. **Answer:** Use RREF to see that $v_4 - v_1 - v_2 = 0$.

- What is the largest number of linearly independent vectors among the $\{v_i\}$. You must write down an explicit set of vectors in S which are independent. **Answer:** The maximum number is 3 or the rank of M. v_1, v_2, v_3 form a matrix of rank 3. Likewise, v_1, v_2, v_5 also works.