Quiz 7 Linear Transformation.

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Consider a linear transformation $T : \Re^3 \to \Re^2$ such that the image of a vector $v \in \Re^3$ is a vector $w \in \Re^2$ which is defined as the column with top entry being the average of the top two entries of v and the bottom entry being the average of the bottom two entries of v.

For example $T(\begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}) = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

Answer the following questions.

- Determine the images by T of the standard basis vectors e_1, e_2, e_3 of $\Re^3$. (You also know these as $\mathbf{i}, \mathbf{j}, \mathbf{k}$.).

  Answer: The images are $T(e_1) = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$, $T(e_2) = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$, $T(e_3) = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix}$.

- Using the above, or otherwise, determine the matrix A of the transformation T, i.e. find the matrix A such that $T(X) = AX$ for any $X \in \Re^3$.

  Answer: Hence $A = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{pmatrix}$.

- For meditation later. Without further calculations, you can decide that the transformation T is not injective (i.e. not one to one).

  Similarly with a little thought, you can deduce that the transformation T is surjective (i.e. onto).

  Think about why.

  Answer: Recall that the map T_A defined by $X \to AX$ is surjective iff $rank(A) = rownum(A)$ and injective iff $rank(A) = colnum(A)$. Since $rank(A)$ is clearly 2, the results follow.