MA322
Sathaye **Topics for Exam 1** **on October 2 2017**

The first exam will six questions (with parts) and will be given during the evening (5-7 PM). You will have two additional pages to record work which does not fit in the original pages.

Work on the back side of all pages is treated as scratch work. It will not be graded. The exams will be scanned, so you must have the correct GUID clearly entered. Please write with a (medium) soft pencil or pen and avoid writing too close to the edge.

Some practice problems are on the web site. However, some might include topics outside the scope of the current exam. Such problems will appear on a later exam.

For the first exam, these are the topics that you should prepare for.

1. **Setup** Convert a system of linear equations to an augmented matrix with a title row. Conversely convert an augmented matrix with a title row to a system of equations.
2. **Solution Process** Convert a matrix to REF or RREF as asked. The **standard algorithm** as discussed in the class and notes should be used for such questions. [1]

   The row operations for each step must be displayed in proper notation.
3. **Reporting solutions** Read off solutions from a given final form of an augmented matrix.

   In case of REF use necessary back substitution and then report the solution in the standard parametric form $X = X_p + s_1 X_1 + \cdots s_r X_r$.

   In case of RREF, you don't need the back substitution, but the final answer should be still in the standard parametric form.
4. **Definitions** Expect questions of the form "Complete the following definition: ". This may be followed by questions which ask you to provide your own examples (or counterexamples) related to the definitions.

   You should also expect questions about a concrete example asking you to decide if the definition is satisfied or fails. Your answer must be justified based on your stated definition.
5. Learn what is meant by **the rank of a matrix.** You should be able to use any of the equivalent definitions:
   - Basic definition: Number of pivots in an REF (or RREF) of the matrix.
   - Determinant definition: Size of the largest non zero subdeterminant of the matrix.
   - In terms of linear independence: Largest number of linearly independent columns of the matrix.
     The pivot columns of an REF give the maximum number of linearly independent columns of the matrix, provided you pick the pivot columns from the original matrix.
6. **Sample Definitions** A list of definitions to study (memorize) includes:
   - $\Re^n$. Independence and dependence of vectors in $\Re^n$.
   - Span of a given set of vectors.
   - Linear transformation T from $\Re^n$ to $\Re^m$. Definition of injective, surjective and bijective linear transformations.
     The matrix A such that $T = T_A$, i.e. matrix of the linear transformation T.
     Produce examples of transformations having desired properties.
   - $Ker(T), Image(T)$ for a linear transformation T. Relation of these with $Nul(A), Col(A)$, when $T = T_A$.

   The above list is only a typical sample. **Important.** Be sure to study the full list of definitions on line (In the Lecture/notes link).
7. Learn the main fact that the rank of a matrix is **less than or equal to** the minimum of its rownum and colnum.
8. Learn why given n columns in $\Re^m$ are dependent if $n > m$ and cannot span $\Re^m$ if $n < m$.

   Be sure to write arguments in complete sentences, rather than just a "one word or symbol" answer.

[1] It is expected that your answers are precise fractions and not approximate decimal numbers from the calculator. If you do use the calculator, you should know how to get the precise answer. It is also possible to teach the calculator to produce precise answers.

9. Given a map T from $\Re^n$ to $\Re^m$ learn how to **prove** if T is a linear transformation.

   You should also be a able to find the matrix A of T in the standard basis and using A, decide if T is injective/surjective etc.

   You should be able to find enough independent vectors in $Ker(T)$ and $Image(T)$. (This is done by solving the appropriate equations.)

10. Practice how to find the **Consistency Matrix** for a given Matrix A. You are expected to know the meaning and uses of the consistency matrix.

    Pay special attention to the case when the consistency matrix of A is empty!

    This happens when $rownum(A) = rank(A)$. This corresponds to

    - $(A|B)$ is consistent for all possible B. Of course the eligible B are from $\Re^m$ where $rownum(A) = m$.
    - T_A is surjective (onto).