DEPARTMENT OF MATHEMATICS

Ma322 - EXAM #3 Fall 2008
November 24, 2008

DO NOT TURN THIS PAGE UNTIL YOU ARE INSTRUCTED TO DO SO.

Be sure to show all work and justify your answers.
There are 4 problems and a total of 5 pages including this one. No other sheets, books, papers are allowed.

Problem	Maximum Score	Actual Score
1	12	
2	12	
3	12	
4	12	
Free	2	
Total	50	

NAME: ______________________________

1. (a) Let A be an $n \times n$ matrix over real numbers.

   - Define an eigenvalue of A.

   - Define an eigenvector of A.

   - Define an eigenspace of a given eigenvalue (say λ) of A. In the lecture, I usually denoted this as V_λ.

   (b) Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 2 \\ 0 & 0 & 5 \end{pmatrix}.$$

   Determine the characteristic polynomial of A and calculate all the eigenvalues of A.

   (c) Determine a basis for the eigenspace belonging to the largest eigenvalue.

2. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}.$$

Determine the characteristic equation and all the eigenvalues and corresponding eigenspaces of A.

Use your calculations to diagonalize A, i.e. to find a matrix P such that $A = PDP^{-1}$ where D is a diagonal matrix.

Answer. $P =$ ________________ $D =$ ________________

3. You are given the following vectors in $\Re^4$.

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, u3 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}.$$

Use the usual inner product.

Answer the following questions.

(a) Find $||u_1||, ||u_2||$.

(b) Find the angle between u_1, u_2.

(c) Find the projection of u_3 in the direction of u_2.

4. You are given the following vectors in $\Re^3$:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, w = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.$$

Use the usual inner product.

(a) Determine if v_1, v_2 are orthogonal.

(b) Find the projection of the vector w into the vector space spanned by v_1, v_2.

(c) Let A be the 3×2 matrix with columns v_1, v_2. Using your calculation above, determine if the equation $AX = w$ has a solution.

Hint: No further calculations are needed.