MA322
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Know these! **0801015**

These are the questions that you should know the answers to. The knowledge will provide a good preparation for the next text.

1. What is a vector space? Define precisely. Give examples of set S with operations $+$ and $\cdot$ such that S is closed under the operations but fails one of the other axioms of the vector space.
2. Give five different examples of vector spaces.
3. What is the definition of a set of independent vectors? Give examples of one, two and five independent vectors with justification.
4. What is meant by the span of a set of vectors? What is meant by the statement that a set of vectors is a spanning set of a vector space?
5. Give example of a spanning set for a vector space. Give example of a set of five vectors in a vector space V which is not a spanning set for V. Justify your claim.
6. When is a set of vectors said to be a basis of a vector space?
7. Construct an example of a vector space V which has following three examples in it.
   - A set of five vectors which forms a basis of V.
   - A set of five vectors which does not form a basis for V. (These five vectors should be non zero and none of them should be a multiple of another.)
   - A set of five vectors which span V.
   - A set of five vectors which do not span V. (These five vectors should be non zero and none of them should be a multiple of another.)
8. What is meant by the dimension of a vector space?
9. Give three distinct examples of an infinite dimensional space.
10. What is the definition of a linear transformation (homomorphism) of vector spaces?
11. Give three examples of linear transformations between infinite dimensional vector spaces.
12. Give example of a map between two vectors spaces which is a linear transformation. Give example of a map between the same two vector spaces which is not a linear transformation.
13. How do you determine if a linear transformation is injective? Give an example of the process.
14. How do you decide if a linear transformation is surjective? Give an example of the process.
15. What is meant by the Kernel and the Image of a linear transformation? Give at least three examples of linear transformations whose kernels and images are non zero vector spaces.
16. What are the two basic ways of describing a subspace of a vector space? (Answer: As a kernel of a linear transformation or as the image of a linear transformation. In $\Re^n$ this takes on the form of $Nul\ A$ and $Col\ A$.) Give exaamples.
17. Given a subspace of the form $Nul\ A$ how do you write it as $Col\ B$ for some B?
18. Given a subspace of the form $Col\ B$ how do you write it as $Nul\ A$ for some A?
19. What is the fundamental dimension formula in vector spaces which relates the dimension of a vector space V, the dimension of the kernel of a linear transformation of V and the dimension of the image of the same transformation?
20. The fundamental dimension formula is related to the number of pivots in the row echelon form of a matrix. What is this connection?
21. If any independent set of vectors is given in a vector space, it can be enlarged to make a basis for the same vector space. Give an example of this process, where at least two extra vectors are needed.

22. Given any spanning set of a vector space, it can be trimmed down to a basis. Give an example of this process where at least two vectors need to be trimmed.

23. Given a vector space V and an ordered basis $B = \begin{pmatrix} v_1 & \cdots & v_n \end{pmatrix}$ what is the definition of the vector $[v]_B$? What is this vector called? Give example of finding such a vector in a finite dimensional space. Give a similar example in an infinite dimensional space.

24. Given a linear transformation $T : V \to W$ and ordered bases B of V and C of W how do you find the matrix of the transformation with respect to these given bases? Answer: It is the matrix M whose columns are $[T(v_1)]_C, [T(v_2)]_C, \cdots, [T(v_m)]_C$ where $B = \begin{pmatrix} v_1 & \cdots & v_m \end{pmatrix}$. Give examples of such calculations when V is $M_{2,2}(\Re)$. Same exercise when $V = P_n$ for some n. Corresponding W can be the same as V or different.

25. Given two ordered bases B and C of the same vector space V, how are they related by a matrix? How are the respective coordinates related by the same matrix? Answer: Write $C = BM$. Then $[v]_B = M[v]_C$ for all vectors v. The matrix M is suggestively denoted as P_B^C and its columns are simply the coordinate vectors of members of C with respect to the basis B.

    Give examples of such calculations in $\Re^n$. Do similar examples in other vectors spaces.

26. What is a determinant? For which matrices A is $\det A$ defined?

27. What are the formulas for a 2×2 and a 3×3 determinant?

28. Which determinants have easy evaluations? List as many as possible. For example: A determinant with a zero column, a determinant where one row is a sum of other two, a triangular determinant etc.

29. What is the best way of evaluating a large determinant (by hand)? Practice.

30. How does a determinant help in determining the rank of a matrix?

31. How does one solve linear equations using determinants? (Cramer's Rule.)

32. What is an adjugate (classical adjoint)? How is it related to the inverse?

33. How is a determinant used to decide if a matrix is invertible?

34. What is the relation between $\det AB$ and $\det A$ and $\det B$?

35. What is the connection between a determinant and the area or volume? Give concrete uses in $\Re^2$ and $\Re^3$ respectively.

36. How is a determinant used to write convenient equations for the following geometric objects?

    - Equation of a line joining two points in the plane. (It is a 3×3 determinant equated to zero.
    - Equation of a plane through three given points. It is a 4×4 determinant equated to zero.

37. How does the general Laplace expansion work? Give example of evaluating a determinant using expansion by two rows.

38. Devise questions of your own to make the number of questions a prime number. How many more would be needed?