

9/17/18 NOTES MA 330

Two Residue Problems:

$$\begin{aligned} n &= ax+r \\ n &= by+s \\ ax-by+r-s &= 0 \\ \text{OR: } ax-by &= c, \quad c=r-s \end{aligned}$$

No Residue Problems:

$$\text{solve } ax-by=d \quad d=\text{gcd}(a,b)$$

For any $c=md$ solve

Always want the general solution

Practice Problem 1:

$$\begin{aligned} n+2 &= 29x & n &= 29x-2 \quad -2=r \\ n-7 &= 45y & n &= 45y+7 \quad 7=s \quad s-r=9 \\ & & 29x-45y &= 9 \end{aligned}$$

126	45	
81	29	1
45	16	1
36	13	1
9	3	4
0	1	3
9	0	

$$\begin{aligned} & -45t \quad -29t \\ & 126(29)-81(45)=9 \\ & 36(29)-23(45)=9 \\ & 29x-45y=9 \\ & 36=x \quad 23=y \\ & n=29(36)-2=1042=45(23)+7 \\ & n=1042+m1305 \\ & \quad \underbrace{\hspace{2cm}} \\ & \quad \text{lcm}(29,45) \end{aligned}$$

Practice Problem 3:

$$\begin{aligned} n &\equiv 5 \pmod{8} & 9-8 &= 1 \\ n &\equiv 4 \pmod{9} & 1(9)-1(8) &= 1 \\ n &\equiv 1 \pmod{7} \end{aligned}$$

$$\begin{aligned} N &= 5+8x & 9y-8x &= 1 & n &= 13 \\ N &= 4+9y & x=1 & y=1 \end{aligned}$$

General $n=13+n(72)$

$$\begin{aligned} n &\equiv 13 \pmod{72} \\ n &\equiv 1 \pmod{7} \end{aligned}$$

$$\begin{aligned} 0 &= -12+7y-72x \\ 12 &= 7y-72x \\ 12 &= 7(12)-72(1) \\ N &= 13+72=85 \end{aligned}$$

31	72	
3	7	10
1	2	3
0	1	2
1	0	

$$\begin{aligned} 31(7)-3(72) &= 1 \\ 372(7)-36(72) &= 12 \\ -72t \quad -7t \quad t &= 5 \\ 12(7)-72(1) &= 12 \end{aligned}$$

Practice Problem 4:

$$\begin{array}{ll} n \equiv 1 \pmod{2} & \text{Let } n-1=m \\ n \equiv 1 \pmod{3} & \text{M is divisible by 2, 3, 4, 5, 6} \\ n \equiv 1 \pmod{4} & \text{Lcm}(2, 3, 4, 5, 6)=60 \\ n \equiv 1 \pmod{5} & \text{M}=60t \\ n \equiv 1 \pmod{6} & N=1+60t = 7s \\ n \equiv 0 \pmod{7} & 7s-60t=1 \end{array}$$

Practice Problem 5:

Same as 4 except $\text{lcm}(2, 3, 4, 5, 6, 7, 8, 9)$
 $=\text{Lcm}(60*7*2*3)$
 $=\text{lcm}(360*7)=2520$

Problem 6 (Revised):

$$\begin{array}{l} n \equiv 2 \pmod{3} \\ n \equiv 3 \pmod{5} \\ n \equiv 5 \pmod{7} \end{array}$$

Find a multiple of $3(5)=15$ which is $1 \pmod{7}$. "15" works

Find a multiple of $5(7)=35$ which is $1 \pmod{3}$. "70" works

Find a multiple of $7(3)=21$ which is $1 \pmod{5}$. "21" works

CLAIM:

$15*5+21*3+70*2$ is an answer.

This works because divided by 3 we get 2, divided by 5 we get 3, and divided by 7 we get 5.