MA 330 Notes 9/21/2018

Practice Problem #7

-The residue of revolutions of Saturn is 24. Find the day number.

-Saturn: In 349,479,375(a) days, it completes 36,641(b) revolutions. Daily motion is 36,641/349,479,375(b/a).

-Use a rational approximation and the problem becomes a number theory problem.

-Saturn is "mean" Saturn, meaning the actual position may not be the same as the prediction.

-24/a is the fraction of the circle of observed Saturn.

Day number: the number of elapsed days since it was at the origin

Origin: All the planets are lined up, taken as the least common multiple of all planet's daily motions. The value is 4,320,000 * 360 days.

36,641x - 24 = ay x=day number, y=completed revolutions

Equation: bx - ay = 24

First, solve: bx – ay = 1

*	394,479,375	
*	36,641	10,766
*	2,369	15
*	1,106	2
45	157	7
2	7	22
1	3	2
0	1	

Now: 24bx – 24ay = 24

-If 24b > a, then reduce mod a

Bhascara I: Wrote a commentary on Aryabhata, bx – ay = 1(Inverse of b mod y), he made tables of these inverses for planets.

-If you observe two planets over the same time and solve the equations, the day number must be the same. If there are differences the planets aren't "mean" planets.

Ptolemi

-2nd century AD, wrote *almagest*

-Used base-60 borrowed from Mesopotamia

-Mapped planetary motion, applied epicycles if there was variation between observations and predictions

Modern Astronomy: Uses trigonometric functions with correction terms

Moon: Equation prepared by Hill, has 70 terms

Archimedes

Key Question: What is the length of a circle?

Strategy: Take regular polygons with bigger and bigger numbers of sides. Find the perimeter. The polygons must fit inside the circle or surround the circle with sides tangent to the circle.

Argument: The circumference is between the perimeter of the inside and outside polygons.

-A limit would give the answer, but limits weren't used.

Zeno's Paradox: If an archer shoots an arrow at a target, it will never reach the target. The arrow can't reach the target until it travels half the remaining distance to the target. Even though this distance becomes very small, it is never zero.

Exhaustion Principle/Dedekind's Theory of Real Numbers

-A real number, x, is defined by two sets L and R

-The union of L and R is all rational numbers

-Also, every "p" in L is less than every "q" in R

-This means L has no maximum because if it had a maximum it would belong to R

Ex) $\sqrt{2}$

$L = \{a \mid a^2 < 2\}$

$R = \{b \mid b^2 \geq 2\}$

$a^2 < 2$ and $b^2 \geq 2$ means that $a^2 < b^2$, does this imply that $a < b$?

Proof

$0 < (b^2 - a^2)$

$0 < (b-a) * (b+a)$

Suppose, if possible, $b-a < 0$ $(b < a)$

If $b > 0$, then the proof obviously fails because the first term is negative, and the second term is positive. The same can be said if $a + b > 0$.

If $a + b < 0$, then $b < -a$