MA 330
Notes for 10/12
Group 4

Find m and n such that $1 + 2 + \dots + m = (m+1) + \dots + n$

$$\frac{m(m+1)}{2} = \frac{n(n+1)}{2} - \frac{m(m+1)}{2}$$

> Similar to house m on a road with n houses such that
>
> 1,2,… (m-1), m, (m+1), … n
>
> sum = sum
>
> and the road had between 50-500 houses. What is n?

$n(n+1) = 2m(m+1)$

$4\,[(n + ½)^2 - ¼ = 2(m + ½)^2 - ¼]$

$(2n+1)^2 - 1 = 2((2m+1)^2 - 1)$
$(2n+1)^2 = 2(2m+1)^2 - 1$
$x^2 - 2y^2 = -1$

(1,1) is a solution but not for n, m.
D = 2
(1, 1, -1) is a 3 triple. $\quad x^2 - 2y^2 = 1 \leftarrow \quad (3,2)$
* (3, 2, 1)

(1, 1, -1) * (3, 2, 1) = (3 + 2(2), 3+2, -1) = (7, 5, -1)

2n+1 = 7	m = 2m+1
n = 3	5 = 2m+1
	m = 2

so, 1 + 2 = 3

(7, 5, -1) * (3, 2, 1) = (21+2(10), 14+15, -1) = (41, 29, -1)

2n+1 = 41	2m+1 = 29
n = 20	m = 14

so, 1 + 2 + 3 +…+ 14 = 15 +…+ 20

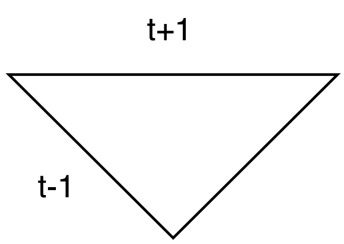
$m + (m+1) + \dots + (n-1) + n = mn$
$$\frac{n(n+1)}{2} - \frac{m(m-1)}{2} = mn$$
$n(n+1) - m(m-1) = 2mn$

$(n+½)^2 - ¼ - (m-½)^2 + ¼ = 2mn$
$4\ [(n+½)^2 - (m-½)^2 = 2mn]$
$(2n-1)^2 - (2m-1)^2 = 8mn$
$4n^2+4n+1-8mn = 4m^2-4m+1$
$4n^2+4n(1-2m)+1$
$(2n+1-2m)^2 - (1-2m)^2 + 1 = 4m^2-4m+1-1$
$(2(n-m)+1)^2 = 2(2m-1)^2 - 1$
$x^2 - y^2 = -1$

same problem/solutions with different m,n combinations

t is an integer

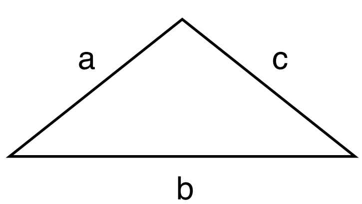
t-1, t, t+1 are side lengths of a triangle and the area is an integer



Prove that $3(t^2-5) = h^2$ with h being even.

$t^2 \equiv 1 \mod 4$
$3(4x^2 - 4) = h$ $2x=t$ *t must be even
Solve $x^2 - 3y^2 = 1$ for some y

Heron's Formula



Set $s = \frac{a + b + c}{2}$ (semi-perimeter)

Area = sqrt[s(s-a)(s-b)(s-c)]

$$s = \frac{t-1+t+t+1}{2} = \frac{3t}{2}$$

$$s - a = \frac{3t}{2} - t + 1 = \frac{t}{2} + 1$$

$$s - b = \frac{3t}{2} - t = \frac{t}{2}$$

$$s - c = \frac{3t}{2} - t - 1 = \frac{t}{2} - 1$$

$$\left(\frac{3t}{2}\right)\left(\frac{t}{2}\right)\left(\frac{t}{2} + 1\right)\left(\frac{t}{2} - 1\right)$$

$$\frac{3t^2}{4}\left(\frac{t^2}{4} - 1\right) = \frac{t^2}{4}\left(3(t^2 - 4)\right)$$

$t \equiv 0 \bmod 4$ or $t \equiv 1 \bmod 4$

$t = 2p + 1$

$t^2 = 4p^2 + 4p + 1$