

continued fractions

$$\frac{13}{8} \rightarrow 1 + \frac{5}{8}$$

$$= 1 + \frac{1}{\frac{8}{5}}$$

$$= 1 + \frac{1}{1 + \frac{3}{5}}$$

$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{2}{3}}}$$

$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$$

continued fractions are denoted by brackets
 $[1, 1, 1, 1, 2]$ → this one is regular because it stops

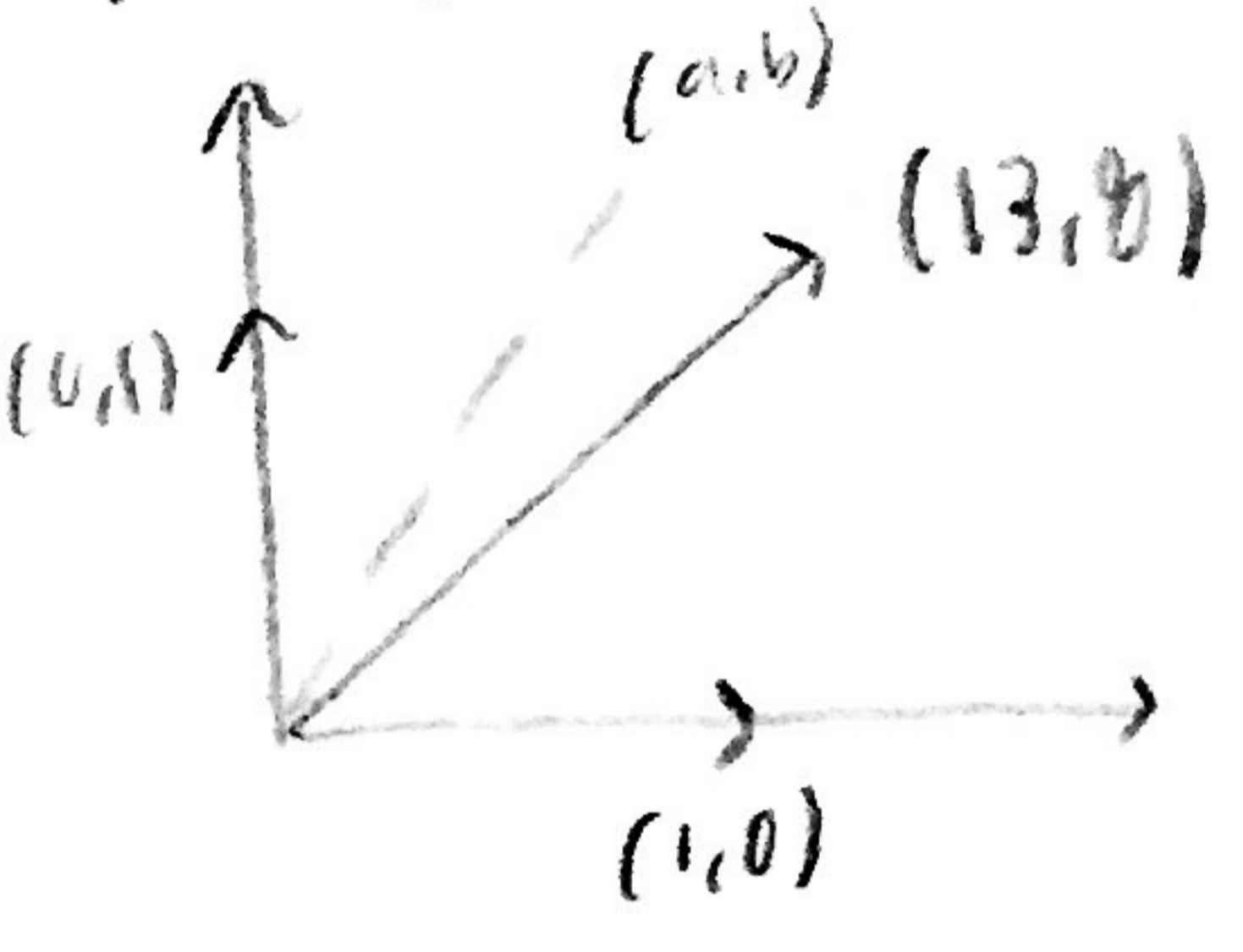
you can start with a fraction and break it down

ex: $\frac{13}{8} \rightarrow \frac{8}{5} \rightarrow \frac{5}{3} \rightarrow \frac{3}{2} \rightarrow \frac{2}{1}$

or you can start at the end and go back up

ex: $2 \rightarrow \frac{1}{2} + 1 = \frac{3}{2}$

you can also think of $\frac{13}{8}$ as a vector in a plane



$$\begin{pmatrix} a \\ b \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} c \\ 1 \end{pmatrix}$$

↳ you want to get really close to (13, 8)

- how can you tell if (a, b) will cross (13, 8)?

calculate the determinant of $\begin{vmatrix} a & 13 \\ b & 8 \end{vmatrix}$

if $\det > 0$ above line
 $\det = 0$ on the line
 $\det < 0$ below line

ex. $\begin{vmatrix} 1 & 13 \\ 0 & 8 \end{vmatrix} = 8 - 0 = 8 > 0$

$\begin{vmatrix} 0 & 13 \\ 1 & 8 \end{vmatrix} = 0 - 13 = -13 < 0$

- you want to add to $\begin{pmatrix} a \\ b \end{pmatrix}$ multiples of $\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} a \\ b \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a+c \\ b \end{pmatrix}$

Q: what values of c makes $\begin{vmatrix} c & 13 \\ 1 & 8 \end{vmatrix} < 0$?