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This period begins with the invention of analytical geometry and the infinitesimal calculus. The mathematics is far more complex than that produced in either of the preceding periods: but it may be generally described as characterized by the development of analysis, and its application to the phenomena of nature.

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ERRATA.

Page 22, line 26. *For 410 read 409—356.*

Page 238, line 18. *For Vieta read Snell.*

Page 338. *Dele* lines 6—10 of footnote.

Page 339, line 15 of note. *For second and third editions read third edition.*

Page 339, line 18 of note. *For Cotes read Pemberton.*

Page 390, line 11. *For should have learnt read might have known.*