

Here are some of the points about exam 2 missed by many students.

1. Part 1:3.

You need to argue that the definition of $f(x)$ makes sense. There is no problem in finding $P = L(x)$. Then you need to argue that a line thru P parallel to \overline{BC} indeed meets the segment \overline{AC} . Some axioms are involved here!

Then you need to argue that there is a unique y such that $Q = R(y)$. This can be deduced from Part 1:2.

2. Part 1:4. You need to show that when $x_1 < x_2$, $f(x_1) < f(x_2)$.

The definition of f involves corresponding points P_1, Q_1 and P_2, Q_2 .

Note that the hypothesis implies $A-P_1-P_2$.

Also, note that $f(x_1) < f(x_2)$ means $A-Q_1-Q_2$.

Thus, you need to show: $A-P_1-P_2$ implies $A-Q_1-Q_2$.

A suitable application of the separation axioms will prove this.

3. Part 1:5. $f(x)$ is one to one means given $x_1 \neq x_2$ we get $f(x_1) \neq f(x_2)$.

This follows from Part 1:4.

To prove that $f(x)$ is onto means given $y \in [0, 1]$, we can find $x \in [0, 1]$ such that $f(x) = y$.

This is done by explicitly showing how the intermediate points Q, P can be found.

4. Part 1:6. Define $L(x_1) = P_1, L(x_2) = P_2$ and note that $L(\frac{x_1+x_2}{2}) = P$ is then the midpoint of P_1, P_2 . (You need some argument with coordinates).

By properties of parallel lines, argue that Q , the corresponding point on \overline{AC} is the midpoint of Q_1, Q_2 .

Now use the fact that $Q = R(y)$ where $y = f(\frac{x_1+x_2}{2})$, to finish off the proof.

5. Part 1:7. Almost nobody proved why p_1, p_2 can be found as stated.

Given $0 < p < 2^{n+1}$ we have two cases. If $p \leq 2^n$, then take $p_1 = 0, p_2 = p$ and check that they satisfy the conditions.

If $2^n < p < 2^{n+1}$, then take $p_1 = 2^n$ and $p_2 = p - p^n$ and argue that they satisfy the condition.

The claim that $u = \frac{p}{2^{n+1}}$ is a good fraction follows from the displayed equation, the averaging property and the induction hypothesis.

6. Part 1:8. Nobody argued the main part of the final argument. It is this:

Suppose, if possible:

$x < f(x)$. Then there is some good fraction $t = \frac{p}{2^n}$ such that $x < t < f(x)$.

Proof. We want: $2^n x < p < 2^n f(x)$. Thus, p can be any integer between $2^n x$ and $2^n f(x)$. Thus, we only need to assure that there is some integer between these two numbers $2^n x$ and $2^n f(x)$.

If the distance between two real numbers is large enough, then it is clear that there is an integer between them. In fact, it is enough to assume that the distance between two real numbers is bigger than 1!

The distance is $2^n(f(x) - x)$ and clearly since $f(x) - x > 0$ we see that multiplying by a sufficiently large 2^n the distance will get above 1.

Done!

Now, given $x < t < f(x)$ with t a good fraction, we get $f(x) < f(t)$ since f is increasing and thus $f(x) < t$ since $f(t) = t$ by goodness of t .

Thus, we have a contradiction: $t < f(x) < t$.

If on the other hand, $x > f(x)$, then by a similar argument, we find a good fraction t with $x > t > f(x)$ and get a similar contradiction.

7. Part 2:1. Here the only problem was that many of you chose to read the treatment in the book rather than my handout. The problem with that approach is that you did not define the symbols used. A more serious problem was that you did not pay attention to the change in the concept to using a signed distance in place of the positive distance that we have been using.

As a result, you got wrong answers caused by misinterpretation!

8. Part 2:4.

Many of you just remembered the words similar triangles used in class, but did not clarify which triangles are similar. In fact, I found evidence that many did not know which triangles were similar.

You drop perpendiculars \overline{AM} , \overline{PN} from A and P onto \overline{BC} respectively. Then you can show the triangles $\triangle AMA_1$, $\triangle PNA_1$ to be similar and conclude that

$$\frac{PN}{AM} = \frac{PA_1}{AA_1}.$$

Moreover $\Delta = \frac{1}{2}AM \cdot BC$ and $\Delta_A = \frac{1}{2}PN \cdot BC$.

From this, it follows that the above ratio is also equal to $\frac{\Delta_A}{\Delta}$.

For the final conclusion, you should use the formulas from my handout for the Barycentric coordinates and deduce that this is the Barycentric coordinate of A , for the point P . The full formula follows by symmetry.