MA361 (Sathaye)
(190213) **Exam 1a Take Home** **Name**________________________

Due on 2/13/19.

Be sure to start each question on a new page and be sure to staple the pages together in correct order at the end.

All answers must carry explanations in words. You should first write what you claim to prove, and then proceed with the proof. Any formulas or theorems you use must be stated clearly, before using.

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1. **Points 15**
   Let G and H be cyclic groups of orders $7, m$ respectively. Let us write e_G as the identity of G and e_H as the identity of H.

   Assume that $G =< a >$ and $H =< b >$. Let $f : G \to H$ be a homomorphism such that $f(a) = b^r$ where $r > 0$ is an integer.

   Answer the following:

   (a) If $m = 25$ show that r must be 0. Explain why this means $f(x) = e_H$ for all $x \in G$.

   (b) If $m = 21$ then show that r must be a multiple of 3. The original 7 was a typo. It is now corrected here and in the next line.

   Moreover, show that there is homomorphism f such that $r = 3$.

   Answer:

2. **Points 15**
   Now assume that A and B are groups. Further, let $|A| = a$ and $|B| = b$ for natural numbers a, b.

   - If $a > b$ then explain why there is no 1-1 homomorphism $g : A \to B$.
   - If $b > a$ then explain why there is no onto homomorphism $g : A \to B$.
   - Suppose $a = b$. Give examples of groups A, B which are not isomorphic.

   Answer:

3. **Points 20**
   Let G be a group and let H be a chosen finite subgroup of G. Define a relation $\equiv_H$ on G as follows:

   If g_1, g_2 are in G, then we say $g_1 \sim_H g_2$ iff $g_1 = g_2 h$ for some $h \in H$.

   Prove the following:

   - $\sim_H$ is reflexive.
   - $\sim_H$ is symmetric.
   - $\sim_H$ is transitive.
   - Thus, $\sim_H$ is an equivalence relation. Show that the number of elements in any equivalence class is equal to $|H|$

   Answer: