Outline for Ma 764 Fall 2014.

Here is a list of topics that I plan to discuss. I can skip the discussion for topics already familiar to the audience, and expand on the topics which are unfamiliar. As a general outline, I would follow my brief article on Affine Geometry in the published obituary in the Notices. I would now list individual topics and details below.

By the beginning of Fall, I would also have some notes of my own, based on the talks on the same themes that I plan to give in Kolkata India in June 2014.

1. Polynomial Rings: Recognizing variables in polynomial rings (also viewed as affine spaces). Main questions about polynomial rings embedded in other polynomial rings. Problems in two and three dimensions.

2. Basics of Curve Theory - affine as well as projective: Algebraic curves in the plane and three space. Concepts of simple and singular points, differentials and genus formulas including the Dedekind formula. The Abhyankar-Sathaye book on "Algebraic. Theory of space curves" may be used for the direct treatment of genus developed by Abhyankar.

3. The rank of an affine curve: I will also introduce and develop the concept of rank of an Affine curve. Abhyankar-Heinzer-Sathaye paper in the book dedicated to Seshadri has formal details. I will give a somewhat simpler treatment. Abhyankar wanted to rewrite Jung's original treatment of a projective surface and was discussing it seriously after the AHS paper, but that plan was only partially finished. I will give some of our discussions.

   I will give some applications of the rank along with its connections with Epimorphism (Suzuki version) as well as the Lin-Zaidenberg Theorem.

4. The Epimorphism Theorem: I will discuss the epimorphism theorem of Abhyankar-Moh including my generalization. This is the topic of curves with one place at infinity and the use of approximate roots. The old TIFR notes by B. Singh is a handy reference for getting familiar. There are other references as well. This theorem, proved in the early 70's has been given alternative proofs using various theories of Algebra and Algebraic Geometry as well as Algebraic Topology (Knot Theory).

5. Curves in positive characteristic: I will discuss some new ideas about one place curves in char. p, developed in discussions with R. Ganong. Ganong's papers are a good source (available on line). Moh's thesis work on the Galois theory in char. p is also useful.

6. Abhyankar-Sathaye Conjecture: Proposed generalizations of Epimorphism Theorem will be discussed at least in three space. The details depend on audience interest!

7. The Jacobian Problem: Finally, I will discuss the Jacobian problem. I will mostly stay with the plane problem using the Newton diagram approach. Abhyankar as well as Moh always felt that the approach should work, except one needs a stronger push and a more detailed analysis. I will certainly describe some of the details from the series of papers written by Abhyankar after 2001, until he moved into investigations of the dicritical divisors.

   The topic of dicritical divisors, though fascinating, needs a more extensive familiarity with Algebraic Geometry. We probably would stop with just a brief description.

   It is possible that the above list is too ambitious for a single semester. However, I promise to continue at least a private seminar depending on the interest.