Directional Derivatives and the Gradient Vector

MA 213 - Worksheet 15

1. Find the directional derivative of f at the given point in the indicated direction.

   (a) $f(x,y) = x\cos(xy)$, $(0,1)$, $\theta = \frac{\pi}{4}$ (b) $g(s,t) = s\sqrt{t}$, $(2,4)$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$

2. Find the gradient of f, evaluate the gradient at the point P, and find the rate of change of f at P in the direction of the vector $\mathbf{u}$.

   (a) $f(x,y) = \dfrac{x}{y}$, $P(2,1)$, $\mathbf{u} = \dfrac{3}{5}\mathbf{i} + \dfrac{4}{5}\mathbf{j}$

   (b) $f(x,y) = f(x,y,z) = x^2yz - xyz^3$, $P(2,-1,1)$, $\mathbf{u} = \left\langle 0, \dfrac{4}{5}, -\dfrac{3}{5} \right\rangle$

3. Suppose that the temperature at a point (x,y,z) in space is given by $T(x,y,z) = 80/(1 + x^2 + 2y^2 + 3z^2)$, where T is measured in degree Celsius and x, y, z in meters. In which direction does the temperature increase fastest at the point $(1,1,-2)$? What is the maximum rate of change?

4. The altitude of a mountain at (x,y) is $f(x,y) = 2500 + 100(x + y^2)e^{-0.3y^2}$ where x, y are in units of 100m.

   (a) Find the directional derivative of f at $P(-1,-1)$ in the direction indicated by $\theta = \pi/4$.

   (b) What is the interpretation of this derivative?

5. Find all points at which the direction of fastest change of the function $f(x,y) = x^2 + y^2 - 2x - 4y$ is $\mathbf{i} + \mathbf{j}$.

6. Are there any points on the hyperboloid $x^2 - y^2 - z^2 = 1$ where the tangent plane is parallel to the plane $z = x + y$?