Homework for Ma 764 - Algebraic Geometry (Fall 02)

Set 1

1. Let K be an infinite field. Show that the affine variety defined by a polynomial $f \in K[z_1, \ldots, z_n]$ is $\mathbb{A}^n$ if and only if $f = 0$.
Is this statement still true if K is a finite field?

2. Let K be an algebraically closed field. Prove that a non-constant polynomial $f \in K[Z_0, \ldots, Z_n]$ is homogenous if and only if the following implication is true:
If $f(a_0, \ldots, a_n) = 0$ for some $(a_0, \ldots, a_n) \in K^{n+1}$ then $f(\lambda a_0, \ldots, \lambda a_n) = 0$ for all $\lambda \in K$.

3. Let $\Gamma \subset \mathbb{P}^n$ be a subset of d points which is not contained in a line. Show that Γ may be defined by polynomials of degree $\leq (d-1)$.

4. Show that any finite subset of a rational normal curve is in linearly general position, i.e., any four of the points span $\mathbb{P}^3$.

Due date: September 11, 2002 (Wednesday)