Homework for Ma 764 - Algebraic Geometry (Fall 02)

Set 2

5. Let $\mathfrak{a}$ be a homogenous ideal in $K[Z_0, \ldots, Z_n]$. Write a polynomial f in the form $f = \sum f_j$ where f_j is a homogeneous polynomial of degree j. Prove that f is in $\mathfrak{a}$ if and only if every homogeneous component f_j is in $\mathfrak{a}$.

6. Show that every non-empty open subset of $\mathbb{P}^n$ over an infinite field is dense, i.e. its closure is $\mathbb{P}^n$.

7. Let $f \in K[z_1, \ldots, z_n]$ be a non-trivial polynomial. Describe explicitly an isomorphism of K-algebras $K[z_1, \ldots, z_n]_f \to K[z_1, \ldots, z_{n+1}]/(1 - z_{n+1} f)$.

8. Let $X \subset \mathbb{P}^n$ be a projective variety and denote by $Y = \nu_d(X) \subset \mathbb{P}^N$ its image under the d-th Veronese map.
(a) Show that X and Y are isomorphic.
(b) What is the relation between the homogeneous coordinate rings of X and Y?

9. Show that every projective variety is isomorphic to an intersection of a Veronese variety and a linear variety. Conclude that any projective variety is isomorphic to an intersection of quadrics.

10. Read Lectures 2 - 4 in Harris' book. Sketch the ideas which are needed in order to prove its Theorem 3.13.
Recall the concept and the properties of primary decompositions.

Due date: October 7, 2002 (Monday)