Homework for Ma 764 - Algebraic Geometry (Fall 02)

Set 5

23. Show that the Hilbert polynomial of a plane curve $C \subset \mathbb{P}^2$ is uniquely determined by its degree. Is the same conclusion still true for curves in $\mathbb{P}^n$ where $n \geq 3$?

24. Let $f \in K[Z_0, \dots, Z_n$ be a homogeneous polynomial. Prove the Euler formula

$$\sum_{i=0}^{n} \frac{\partial f}{\partial Z_i} \cdot Z_i = \deg f \cdot f.$$

25. Show that every rational normal curve $C \subset \mathbb{P}^3$ is smooth.
(Hint: You may assume that the homogeneous ideal of C is

$$I_C = (Z_0 Z_2 - Z_1^2, Z_0 Z_3 - Z_1 Z_2, Z_1 Z_3 - Z_2^2).$$

26. Let $C \subset \mathbb{P}^2$ be a smooth curve and let $S \subset \mathbb{P}^3$ be the scheme defined by the ideal $I_C \subset K[Z_0, Z_1, Z_2]$ in the (larger) ring $K[Z_0, \dots, Z_3]$. Determine the dimension and the singular points of S. Describe S geometrically.
(Hint: Consider a line through any point of C and the point $(0 : 0 : 0 : 1)$.)

Due date: November 27, 2002 (Wednesday)