

### Worksheet 3 for MA 113 - Calculus I (Spring 08)

02/12/08

This worksheet discusses how many tangent lines to the parabola pass through any given point  $(c, d)$  (see the sketch below). The exercise below answers this question and allows you to relate the number of tangent lines to the location of the point.

*As always, write up your solutions neatly, carefully, and in complete sentences.*

Before beginning, it might be helpful to recall the quadratic formula. The roots of the quadratic equation  $ax^2 + bx + c = 0$  are

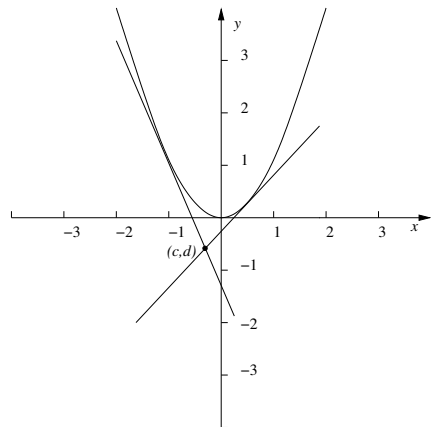
$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quantity inside the radical,  $b^2 - 4ac$ , is called the *discriminant*. It follows that we have two real roots if the discriminant is positive, one real root if the discriminant is 0, and no real roots if the discriminant is negative.

1. Consider the parabola  $y = x^2$ .
  - (a) Write the equation of the tangent line to the parabola  $y = x^2$  at  $(a, a^2)$ .
  - (b) Find all tangent lines to the parabola  $y = x^2$  that pass through the point  $(2, 3)$  (see also the hint for part (e)).
  - (c) For each of the points below, make a sketch which shows the parabola given by  $y = x^2$  and all tangent line(s) to this parabola which pass through the specified point.
    - (i)  $(0, 1)$
    - (ii)  $(1, -2)$
    - (iii)  $(1, 1)$ .

Now let  $(c, d)$  be a point which may or may not lie on the parabola.

- (d) Make a conjecture as to how many tangent lines of the parabola pass through a given point  $(c, d)$ . How does the answer depend on the point  $(c, d)$ ?
- (e) Give conditions on  $c$  and  $d$  which tell us whether we have exactly 0, 1 or 2 tangent lines through  $(c, d)$ . (Hint: If we require the tangent line in part (a) to pass through point  $(c, d)$ , we obtain an equation for  $a$ . Write out this equation and solve for  $a$ . Give conditions on  $c$  and  $d$  which tell us whether we have 0, 1 or 2 solutions to this equation.)
- (f) Interpret your answers in (e) geometrically. What do these conditions tell us about the location of the point  $(c, d)$  with respect to the parabola?



2. (1 bonus point) The son of the professor's father is talking to the father of the professor's son. Is the professor necessarily speaking? Explain.

**Due date:** February 20, 2008, at the beginning of the lecture