

Worksheet 5 for MA 113 - Calculus I (Spring 08)

03/18/08

Work the following two problems related to curve sketching.

As always, write up your solutions neatly, carefully, and in complete sentences.

1. Carry out the following steps to sketch the graph of

$$f(x) = \frac{x}{1+x^2}.$$

- Find the local maxima and minima of f . Compute the local maximum and minimum values. Give the intervals of increase and decrease.
- Find the inflection points of f . Give the intervals where f is concave upward and where f is concave downward.
- Determine if f is even or odd.
- Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.
- Make a careful sketch of the graph of f that reflects the above information.

For the following problem, it may be helpful to recall that a quadratic equation $ax^2+bx+c=0$ has two distinct roots if the *discriminant* b^2-4ac is positive, one root if the discriminant is zero, and no real roots if the discriminant is negative.

2. Let t be a real number and consider the cubic polynomial $f(x) = x^3 + tx^2 + 3x$.
- Find $f'(x)$.
 - Find the value(s) of t for which f has exactly one critical number.
 - Find the value(s) of t for which f has two critical numbers.
 - Find the value(s) of t for which f has no critical numbers.
 - Draw a sketch of the graph of the polynomial when $t = 3$ and verify that your sketch agrees with your answers to (b) - (d).

Due date: March 26, 2008, at the beginning of the lecture