Homework #1 - Elementary Modern Algebra I (Fall 07)

08/24/07

Please, write down your solutions neatly and explain your reasoning clearly.

1. (4 points) Determine whether the following maps are bijective, and if so, find the inverse map:
(i) $\varphi : \mathbb{N}_0 \to \mathbb{Z}, x \mapsto x^6$;
(ii) $\psi : \mathbb{Z} \to \mathbb{Z}, x \mapsto x^6$;
(iii) $\sigma : \mathbb{Q} \to \mathbb{Q}, x \mapsto \frac{2x+8}{5}$.

2. (4 points) Let M and N be sets with k and l elements, respectively. Determine the number of possible maps from M to N.

3. (4 points) Let $\varphi : M \to N$ and $\psi : N \to P$ be maps. Show that:
(a) If $\psi \circ \varphi$ is surjective, then ψ must be surjective;
(b) If $\psi \circ \varphi$ is injective, then φ must be injective.

4. (4 points) (a) Find an example of maps $\varphi : M \to N$ and $\psi : N \to P$ such that $\psi \circ \varphi$ is surjective, but φ is not surjective.
(b) Find an example of maps $\varphi : M \to N$ and $\psi : N \to P$ such that $\psi \circ \varphi$ is injective, but ψ is not injective.

Due date: August 31, 2007