

## Homework #10 - Elementary Modern Algebra I (Fall 07)

11/12/07

*Please, write down your solutions neatly and explain your reasoning clearly.*

1. (4 points) Let  $R$  and  $S$  be rings. Show:
  - (a)  $R \times S$  is a ring.
  - (b)  $R \times S$  is a commutative ring if and only if  $R$  and  $S$  are commutative rings.
2. (2 points) Let  $R$  and  $S$  be subrings of the ring  $R'$ . Argue that  $R \cap S$  is a subring of  $R'$ , too.
3. (6 points) Let  $R$  be a subring of the commutative ring  $R'$  and let  $a \in R'$  be any element. Prove:
  - (a) The set

$$R[a] := \left\{ \sum_{j=0}^n r_j a^j \mid n \in \mathbb{N}_0, r_j \in R \right\}$$

is a subring of  $R'$ .

- (b) If  $S$  is any subring of  $R'$  such that  $R \cup \{a\} \subset S$ , then  $R[a]$  is a subring of  $S$ , i.e.  $R[a]$  is the smallest subring of  $R'$  that contains  $R$  and  $a$ .
- (c) In  $\mathbb{R}$  we have

$$\mathbb{Z}[\sqrt[3]{2}] = \left\{ a_2(\sqrt[3]{2})^2 + a_1\sqrt[3]{2} + a_0 \mid a_0, a_1, a_2 \in \mathbb{Z} \right\}.$$

**Due date:** November 19, 2007