

Homework #11 - Elementary Modern Algebra I (Fall 07)

11/26/07

Please, write down your solutions neatly and explain your reasoning clearly.

1. (4 points) Let m, n be positive integers.
 - (a) Argue that the map $\varphi : \mathbb{Z} \rightarrow \mathbb{Z}_m \times \mathbb{Z}_n$, $a \mapsto (a \bmod m, a \bmod n)$ is ring homomorphism.
 - (b) Show that the rings \mathbb{Z}_{mn} and $\mathbb{Z}_m \times \mathbb{Z}_n$ are isomorphic if and only if m and n are relatively prime.
2. (4 points) Find all possible ring homomorphisms φ between the indicated rings.
 - (a) $\varphi : \mathbb{Z}_5 \rightarrow \mathbb{Z}_{10}$
 - (b) $\varphi : \mathbb{Q}[\sqrt{2}] \rightarrow \mathbb{Q}[\sqrt{3}]$ (You may use (without proof) the fact that $\sqrt{2} \notin \mathbb{Q}[\sqrt{3}]$).

Due date: December 3, 2007