Homework #6 - Elementary Modern Algebra I (Fall 07)

10/08/07

Please, write down your solutions neatly and explain your reasoning clearly.

1. (4 points) Let G be a group with identity e and let $a \in G$ be an element of order n. Let k be any integer. Argue that $a^k = e$ if and only if n divides k.

2. (4 points) Let $\sigma_1, \ldots, \sigma_k \in S_n$ be disjoint cycles. Show that the order of $\sigma_1 \cdot \ldots \cdot \sigma_k$ is the least positive integer m such that $|\sigma_i|$ divides m for all $i = 1, \ldots, k$. (This number m is called the *least common multiple* of $|\sigma_1|, \ldots, |\sigma_k|$.)

3. (4 points) Find the cycle decomposition, the sign, and the order of the following permutations: $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$;

$$\varphi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 2 & 9 & 7 & 5 & 4 & 3 & 10 & 6 \end{pmatrix}.$$

Due date: October 15, 2007