Homework #9 - Elementary Modern Algebra I (Fall 07)

11/29/07

Please, write down your solutions neatly and explain your reasoning clearly.

1. (4 points) Let $n \in \mathbb{Z}$ be a positive integer. Let a be any integer and set $H := n\mathbb{Z}$. The coset $a + H$ contains a unique smallest non-negative integer b that is called the *canonical representative* of the coset $a + H$.
(a) Argue that $a + H = b + H$.
(b) Show that the canonical representative of $a + H$ is the number $a \bmod n \in \mathbb{Z}_n$.

2. (4 points) Let N be a normal subgroup of the group G. Prove:
(a) If G is abelian then G/N is abelian, too.
(b) If G is cyclic then G/N is cyclic, too.

3. (4 points) Let $\varphi : \mathbb{Z}_{15} \to G$ be a group homomorphism. Show:
(a) If φ is surjective then G is isomorphic to one of the groups $\mathbb{Z}_1, \mathbb{Z}_3, \mathbb{Z}_5$, or $\mathbb{Z}_{15}$.
(b) If G is one of the groups $\mathbb{Z}_1, \mathbb{Z}_3, \mathbb{Z}_5$, or $\mathbb{Z}_{15}$ then describe explicitly a surjective group homomorphism $\mathbb{Z}_{15} \to G$ in each of the four cases.

Due date: November 12, 2007