

Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes nor calculators may be used. When answering the questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer
(*unsupported answers to Questions 1 - 5 may receive NO credit*).

QUESTION	SCORE	TOTAL
1.		20
2.		15
3.		15
4.		20
5.		20
6.		10
TOTAL		100

1. Let $A = \begin{bmatrix} 1 & 0 & -2 \\ -8 & 1 & 2 \\ 9 & -2 & 45 \end{bmatrix}$. Compute the inverse of A if it exists.

2. Find the determinant of the matrix $A = \begin{bmatrix} 4 & 0 & 0 & -2 \\ 0 & -2 & 1 & 0 \\ 0 & 7 & 8 & 9 \\ 0 & 3 & -2 & 0 \end{bmatrix}$.

3. Let $A = \begin{bmatrix} 1 & 3 & 3 \\ 3 & 9 & 9 \\ 2 & 6 & 5 \end{bmatrix}$. Find a basis of $\text{Nul } A$.

4. Consider the set $H = \text{Span}\{v_1, v_2, v_3\}$ where $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ 7 \\ 15 \end{bmatrix}$, $v_3 = \begin{bmatrix} 4 \\ 2 \\ 12 \end{bmatrix}$. Find a basis of H .

5. Argue whether the following sets are vector spaces:

(a) The set of functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $f(5) = 1$.

(b) The set of $m \times n$ matrices.

6. Decide if the following statements are true or false by checking the corresponding box.

true

false

- | | | |
|--------------------------|--------------------------|--|
| <input type="checkbox"/> | <input type="checkbox"/> | If one entry of a square matrix is zero then the matrix is not invertible. |
| <input type="checkbox"/> | <input type="checkbox"/> | If the columns of an $n \times n$ matrix span \mathbb{R}^n then its determinant is not zero. |
| <input type="checkbox"/> | <input type="checkbox"/> | The set of differentiable functions $f : \mathbb{R} \rightarrow \mathbb{R}$ with $f(0) = 0$ forms a vector space. |
| <input type="checkbox"/> | <input type="checkbox"/> | In \mathbb{R}^n every set of less than n vectors is linearly independent. |
| <input type="checkbox"/> | <input type="checkbox"/> | If the the columns of the $n \times n$ matrix A span \mathbb{R}^n , then every subset of the columns of A is linearly independent. |