Homework for Ma 561 - Modern Algebra I (Fall 03)

Set 3

8. (2 points) Let R be an integral domain that has finitely many elements. Show that R is a field.

9. (4 points) (a) Compute $[\mathbb{Q}(\sqrt[3]{2} : \mathbb{Q}]$.
(b) Prove that $X^3 - 2$ is the minimal polynomial of $\sqrt[3]{2}$ over $\mathbb{Q}$.
(Hint: You may use (without proof) the fact that $\sqrt[3]{4}$ is irrational.)

10. (4 points) Let K be a field such that $1 + 1 \neq 0$. Let $a, b \neq 0$ be any elements of K. Show that $K(\sqrt{a}) = K(\sqrt{b})$ if and only if there is an element $c \in K$ such that $a = bc^2$.

11. (4 points) Compute

$$[\mathbb{Q}(\sqrt{2}, \sqrt{3}) : \mathbb{Q}] \quad \text{and} \quad [\mathbb{Q}(\sqrt{2} + \sqrt{3}) : \mathbb{Q}].$$

Is there a relation between $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ and $\mathbb{Q}(\sqrt{2} + \sqrt{3})$?

Due date: September 26, 2003