

Homework for Ma 561 - Modern Algebra I (Fall 03)

Set 4

12. (2 points) Let E/K be a field extension and let F be the algebraic closure of K in E . Show that every $\alpha \in E$ that is algebraic over F is already an element of F (i.e. the algebraic closure of F in E is just F).

13. (3 points) (a) Let E/K be a field extension of degree m . How many elements has E if K has q elements?

(b) Is there a field with 63 elements?

14. (4 points) Let I, J be ideals of the ring R and let $\pi : R \rightarrow R/I$ be the canonical projection. Show:

(a) $\pi(J)$ is an ideal in R/I . It is denoted by $J + I/I$.

(b) The projection π induces an inclusion-preserving bijection between the set of ideals in R that contain I and the set of ideals in R/I .

15. (4 points) Let $\varphi : R \rightarrow R'$ be a ring homomorphism and let $I \subset R$ be an ideal such that $I \subset \ker \varphi$. Prove:

(a) There is a unique ring homomorphism $\psi : R/I \rightarrow R'$ such that $\varphi = \psi \circ \pi$ where $\pi : R \rightarrow R/I$ is the canonical projection.

(b) If φ is surjective then $\ker \psi \cong \ker \varphi/I$, thus ψ induces a ring isomorphism

$$(R/I) / (\ker \varphi/I) \cong R / \ker \varphi.$$

Due date: October 13, 2003