

Homework for Ma 661 - Modern Algebra II (Spring 04)

Set 12

43. (4 points) Decide which of the following polynomials in $\mathbb{Q}[X]$ is separable:

$$X^5 - 2X^3 + 3X^2 - 6,$$

$$X^3 + 5X^2 + 8X + 4,$$

$$X^{72} + 12X^{19} + 10.$$

44. (4 points) Let K be a field of characteristic $p > 0$. Show:

(a) If $f \in K[X]$ is irreducible then f is separable if and only if $f \notin K[X^p]$.

(b) K is perfect if and only if $K = K^p$.

45. (4 points) Let E/K be a field extension. Show that for every algebraic $\alpha \in E$ the following conditions are equivalent:

(a) α is separable over K ;

(b) the minimal polynomial of α over K is a separable polynomial;

(c) $K(\alpha)/K$ is a separable field extension.

46. (4 points) Prove that $[\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}) : \mathbb{Q}] = 8$.

8*. (4 points extra credit) Let E/K be an algebraic field extension. Show that the separable closure E_s of K in E is a subfield of E .

Due date: January 30, 2004