

Homework for Ma 661 - Modern Algebra II (Spring 04)

Set 17

62. (6 points) Let G be a group. Prove Lemma 22.10, i.e. show:

- (a) The commutator subgroup G' of G is a normal subgroup of G .
- (b) If N is a normal subgroup of G and G/N is abelian then $G' \subset N$.
- (c) If $N < G$ and $G' < N$ then N is a normal subgroup of G and G/N is abelian.

63. (4 points) Let N be any positive integer.

- (a) Show that the order of $G(X^n - 5, \mathbb{Q})$ divides $n \cdot \varphi(n)$.
- (b) In case that the order of $G(X^n - 5, \mathbb{Q})$ is $n \cdot \varphi(n)$ describe all automorphisms in $G(X^n - 5, \mathbb{Q})$.

64. (6 points) In the symmetric group S_n , $n \geq 3$, consider the two cycles $\pi := (1, 2)$, $\sigma := (1, 2, \dots, n)$ and the permutation τ where $\tau := (2, n)(3, n-1) \dots (\frac{n}{2}, \frac{n+4}{2})$ if n is even and $\tau := (2, n)(3, n-1) \dots (\frac{n+1}{2}, \frac{n+3}{2})$ if n is odd. Show:

- (a) The subgroup of S_n generated by π and σ is S_n .
- (b) The subgroup of S_n generated by τ and σ is isomorphic to the dihedral group D_{2n} .

Due date: March 12, 2004