

Homework for Ma 661 - Modern Algebra II (Spring 04)

Set 20

73. (4 points) Let R and S be noetherian rings. Show that the product ring $R \times S$ is noetherian, too.

74. (4 points) Let

$$0 \rightarrow V_s \rightarrow \dots \rightarrow V_1 \rightarrow V_0 \rightarrow 0$$

be an exact sequence of finitely generated vector spaces over the field K . Prove

$$\sum_{i=0}^s (-1)^i \dim_K V_i = 0.$$

(Hint: Reduce to the case of a short exact sequence.)

75. (4 points) Prove the characterizations of a noetherian R -module as given in Proposition 26.13.

15*. (4 points extra credit) Let K be an infinite field and let $V \subsetneq \mathbb{A}_K^n$ be an affine variety. Show:

(a) If $n \geq 1$ then $\mathbb{A}_K^n \setminus V$ has infinitely many points.

(b) If K is algebraically closed, $n \geq 2$, and $V \neq \emptyset$ is a *hypersurface*, i.e. V is defined by one equation, then V has infinitely many points.

Due date: April 16, 2004