

Homework for Ma 661 - Modern Algebra II (Spring 04)

Set 21

76. (4 points) Let $W, W_1, W_2 \subset \mathbb{A}^n$ be affine varieties. Show:

- (a) $W_1 \subsetneq W_2$ if and only if $I(W_2) \subsetneq I(W_1)$.
- (b) $V(I(W)) = W$.
- (c) $I(W_1 \cup W_2) = I(W_1) \cap I(W_2)$.

77. (4 points) Let I, J be ideals of the ring R . Prove:

- (a) The *radical* of I , $\sqrt{I} := \{r \in R \mid r^m \in I \text{ for some } m \in \mathbb{N}\}$, is an ideal of R .
- (b) $\sqrt{I \cap J} = \sqrt{I} \cap \sqrt{J}$.

78. (4 points) Let

$$0 \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow 0$$

be an exact sequence of R -modules. Show that

$$\text{Ann}_R M' \cdot \text{Ann}_R M'' \subset \text{Ann}_R M \subset \text{Ann}_R M' \cap \text{Ann}_R M''.$$

79. (4 points) Find for some ring R an example of two R -algebras S_1 and S_2 that are isomorphic as R -modules, but not isomorphic as rings.

16*. (4 points extra credit) Let M be a finitely generated module over the noetherian ring R . Prove:

- (a) There is an exact sequence of R -modules

$$F \xrightarrow{\varphi} G \rightarrow M \rightarrow 0$$

where F, G are free R -modules of finite rank.

- (b) There is a $m \times n$ matrix A with entries in R such $M \cong \text{coker } \psi$ where $\psi : R^n \rightarrow R^m$ is the homomorphism defined by $\psi(v) = A \cdot v$. (A is called a *presentation matrix* of M .)

Due date: April 23, 2004