

## Homework for Ma 561 - Modern Algebra I (Fall 09)

### Set 1

- (2+2 points) (a) Argue that  $\angle\{0, 1\} = \angle\mathbb{Q}$ .  
(b) Show that every subfield of the complex numbers contains the rational numbers.
- (4 points) (Golden ratio) Let  $P \neq Q$  be two points in  $\mathbb{C}$ . Let  $X$  be the point that divides the line segment  $\overline{PQ}$  such that

$$\frac{|PQ|}{|PX|} = \frac{|PX|}{|XQ|}.$$

Show that  $X \in \angle\{P, Q\}$ .

- (2+2+2 points) Consider the set

$$K := \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}.$$

- (a) Show that  $K$ , with the addition and multiplication of  $\mathbb{C}$  restricted to  $K$ , is a field.
- (b) Argue that the map

$$\mathbb{Q} \times K \rightarrow K, \quad (\lambda, x) \mapsto \lambda x,$$

defines a scalar multiplication on  $K$ , thus  $K$  becomes a vector space over  $\mathbb{Q}$ .

- (c) Prove that  $K$  is a 2-dimensional vector space, i.e.

$$\dim_{\mathbb{Q}} K = 2.$$

(Hint: You may use the fact (without proof) that  $\sqrt{2}$  is not a rational number.)

**Due date:** September 4, 2009, at the beginning of lecture.