Homework for Ma 561 - Modern Algebra I (Fall 09)

Set 11

39. (3 points) Show that the following polynomials are irreducible in the respective ring:
(a) $2X^4 + 200X^3 + 40X^2 + 2000X + 20 \in \mathbb{Q}[X]$,
(b) $(Y + 8)^2X^3 - X^2 + (Y + 7)(Y + 8) - Y - 12 \in \mathbb{Q}[X, Y]$,
(c) $X^2Y + XY^2 - X - Y + 1 \in \mathbb{Q}[X, Y]$.

40. (4 points) Determine a factorization by irreducible elements in $\mathbb{Q}[X]$ of the following polynomials:
(a) $X^4 + X + 1$
(b) $X^4 + X^2 + 1$
(c) $X^3 - X^2 - X - 2$
(d) $X^{18} - X^{17} - 6X^3 + 6X^2 + 12X - 12$.

41. (4 points) Show that:
(a) $2 + i$ is irreducible in $\mathbb{Z}[i]$.
(b) $Q(\mathbb{Z}[i]) = \mathbb{Q}(i)$.
(c) $X^n - 2 - i$ is irreducible in $\mathbb{Q}(i)[X]$ for every positive integer n.

42. (4 points) Let $\varphi : K \to K'$ be a ring homomorphism, where K, K' are fields. Let P, P' be the prime fields of K and K', respectively. Prove that:
(a) There is a unique isomorphism $\tau : P \to P'$.
(b) φ is an extension of τ, i.e., $\varphi(a) = \tau(a)$ for all $a \in P$.

Due date: November 20, 2009