Quiz #8

Directions: Carefully read each question below and answer to the best of your ability in the space provided. Your answer to problems should be written in a clear and concise manner.
You **MUST** show your work to receive full credit!

1. (5 points) Find the images of $\vec{u} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ under the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $T(\vec{x}) = A\vec{x}$ with $A = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$.

Solution:

$$T(\vec{u}) = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \end{bmatrix} = \begin{bmatrix} 8 \\ 19 \end{bmatrix}$$
$$T(\vec{v}) = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

2. (5 points) For the following linear transformations find the corresponding 2×2 matrix.

Solution:

1. Dilation by a factor of 3
$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
2. Rotation by 45-degrees clockwise
$$R_{-45^\circ} = \begin{bmatrix} \cos(-45^\circ) & -\sin(-45^\circ) \\ \sin(-45^\circ) & \cos(-45^\circ) \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$
3. Identity matrix
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
4. Rotation by 60-degrees counterclockwise
$$R_{60^\circ} = \begin{bmatrix} \cos(60^\circ) & -\sin(60^\circ) \\ \sin(60^\circ) & \cos(60^\circ) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$
5. Projection onto the x-axis
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Hint: Recall that a general form of the rotation matrix R_α is

$$R_\alpha = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

where $\alpha > 0$ would correspond to the rotation counterclockwise.

Name: _____

Section (circle one): 001 002

Question:	1	2	Total
Points:	5	5	10
Score:			