Quiz #9

Directions: Carefully read each question below and answer to the best of your ability in the space provided. Your answer to problems should be written in a clear and concise manner.
You **MUST** show your work to receive full credit!

1. (5 points) Find the eigenvalues of the following matrix:

$$A = \begin{bmatrix} 2 & \frac{3}{2} \\ 2 & 0 \end{bmatrix}$$

Solution:

$$\det(A - \lambda I) = 0$$

$$\begin{aligned} \det\left(\begin{bmatrix} 2 & \frac{3}{2} \\ 2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) &= \det\left(\begin{bmatrix} 2-\lambda & \frac{3}{2} \\ 2 & -\lambda \end{bmatrix}\right) \\ &= (2-\lambda)(-\lambda) - 2(\frac{3}{2}) \\ &= \lambda^2 - 2\lambda - 3 \\ &= (\lambda - 3)(\lambda + 1) \end{aligned}$$

Therefore, A has 2 eigenvalues $\boxed{\lambda_1 = 3}$ and $\boxed{\lambda_2 = -1}$.

2. (5 points) Find the least squares solution to the following system:

$$\begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Solution:

1. Find $A^T A x = A^T y$ that is

$$\begin{bmatrix} 1 & 1 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$\text{or} \quad \begin{bmatrix} 3 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

Hence the least squares solution is

$$\boxed{\hat{x}_1 = \frac{7}{3}} \quad \text{and} \quad \boxed{\hat{x}_2 = \frac{3}{4}}.$$

Name: ____

Section (circle one): 001 002

Question:	1	2	Total
Points:	5	5	10
Score:			