Math 671 Complex Analysis
Spring 2018
Formula sheet

- Cauchy-Riemann equations:
$$u_x = v_y, \quad u_y = -v_x.$$
- Radius of convergence:
$$R = \liminf_{n\to+\infty} |a_n|^{-1/n}.$$
- Winding number:
$$n(\gamma; z_0) = \frac{1}{2\pi i}\int_\gamma \frac{1}{z-z_0}dz.$$
- Cauchy's residue formula:
$$\frac{1}{2\pi i}\int_\gamma f(z)dz = \sum_{j=1}^{n} \mathrm{Res}(f; z_j).$$
- Cauchy integral formula:
$$f^{(n)}(z_0) = \frac{n!}{2\pi i}\int_\gamma \frac{f(z)}{(z-z_0)^{n+1}}dz.$$
- The argument principle:
$$\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}dz$$
is the total number of zeros inside γ (counted with multiplicity) minus the total number of poles inside γ (counted with multiplicity).
- Rouche's theorem: If $|g(z)| < |f(z)|$ for all z on the curve γ, then f and $f+g$ have the same number of zeros in the interior of γ.
- Laurent series expansion:
$$f(z) = \sum_{n=-\infty}^{\infty} a_n(z-z_0)^n,$$
where
$$a_n = \frac{1}{2\pi i}\int_\gamma \frac{f(z)}{(z-z_0)^{n+1}}dz.$$