STA 291  
Lecture 24

• Two kinds of Error in Testing hypothesis
• Examples.

About **bonus** project
Due in Lab  April 20- 22
• Some survey show a majority people believe in “hot hand”. A follow-up question then is: if there were “hot hand”, how much better/worse a shooter can become by the previous shoots? (i.e. what is a reasonable difference to expect)
• i.e. whether he made or missed the two previous shots, how much difference do you think this has the effect on the present shoot? (in terms of hitting percentages) 5%? 10%? or even 20%?

• Since a small difference will need more data to detect.
• A larger difference can be discovered with less data.
• Some clue: margin of error calculations.
• Bonus is worth equivalent to one LAB

• Lab will start to give “practical quiz”

Decisions and Types of Errors in Tests of Hypotheses

• Terminology:
  – The alpha-level (significance level) is a threshold number such that one rejects the null hypothesis if the p-value is less than or equal to it. The most common alpha-levels are 0.05 and 0.01
  – The choice of the alpha-level reflects how cautious the researcher wants to be (when it come to reject null hypothesis)

Type I and Type II Errors

• Type I Error: The null hypothesis is rejected, even though it is true.
• Type II Error: The null hypothesis is not rejected, even though it is false.

• Setting the alpha-level low protect us from type I Error. (the probability of making a type I error is less than alpha)
Type I and Type II Errors

- Terminology:
  - \( \text{Alpha} \) = Probability of make a Type I error
  - \( \text{Beta} \) = Probability of make a Type II error
  - \( \text{Power} \) = 1 – Probability of a Type II error = 1 - Beta
- For a given data, the smaller the probability of Type I error, the larger the probability of Type II error and the smaller the power
- i.e. If you set alpha very small, it is more likely that you fail to detect a real difference (larger Beta).

- When sample size(s) increases, both error probabilities could be made to decrease.
- Our Strategy:
  - keep type I error probability small by pick a small alpha.
  - Increase sample size to make Beta small.
- Depend on how expensive to obtain data, a Beta = 0.15 is not uncommon.
Type I and Type II Errors

• In practice, alpha is specified, and the probability of Type II error could be calculated, but the calculations are usually difficult (sample size calculation)

• How to choose alpha?
  • If the consequences of a Type I error are very serious, then choose a smaller alpha, like 0.01.
  • For example, you want to find evidence that someone is guilty of a crime.
  • In exploratory research, often a larger probability of Type I error is acceptable (like 0.05 or even 0.1)

Example: New drug development

• The null hypothesis usually state that the new drug is “no difference” to the placebo.

• A type I error in this context is: falsely conclude a drug is useful when it is actually “NO effect”

• A type II error in this context is: falsely dismiss a useful drug.

Alternative and p-value computation

\[ H_0 : p = p_0 \]

<table>
<thead>
<tr>
<th>Alternative Hypothesis</th>
<th>One-Sided Tests</th>
<th>Two-Sided Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_A : p &lt; p_0 )</td>
<td>( p(Z &lt; z_{obs}) )</td>
<td>( 2 \cdot p(Z &gt;</td>
</tr>
<tr>
<td>( H_A : p &gt; p_0 )</td>
<td>( p(Z &gt; z_{obs}) )</td>
<td>( 2 \cdot p(Z &gt;</td>
</tr>
<tr>
<td>( H_A : p \neq p_0 )</td>
<td>( 2 \cdot p(Z &gt;</td>
<td>z_{obs}</td>
</tr>
</tbody>
</table>

\[ z_{obs} = \sqrt{\frac{p - p_0}{p_0(1 - p_0)/n}} \]
Example

• Two consumer products (shampoo, laundry detergent etc) comparison. Call them A vs. B

• n consumers are given both products in the identical packaging. After one week of use of both products, state a preference.

• If there were no difference, then we should see 50%-50%

Suppose in n=236 consumers, 110 prefer product A. Let \( p = \) popu. proportion prefer A. Use \( \alpha = 0.05 \)

• Null: \( H_0: p = 0.5 \)

• Alternative: \( H_A: p \neq 0.5 \)

• Compute \( z_{\text{obs}} \)

Sample proportion, \( \hat{p} = \frac{110}{236} = 0.4661 \)

\[
\frac{\hat{p} - 0.5}{\sqrt{0.5(1-0.5)/236}} = \frac{0.4661 - 0.5}{\sqrt{0.5(1-0.5)/236}} = -1.04156
\]

Finally look the Z table for P-value:
• P-value = \( 2P(Z > 1.04) = 2(1 - 0.8508) = 0.2984 \)
• Conclusion, we do not reject null hypothesis since P-value is not less than alpha.
• Since 0.2984 is not less than 0.05

Two sample cases are similar, with two differences:
• Hypothesis involve 2 parameters from 2 populations
• Test statistic, $z_{\text{obs}}$, is different, involve 2 samples

Alternative and p-value computation

\[ H_0 : p_1 - p_2 = 0 \]

<table>
<thead>
<tr>
<th>One-Sided Tests</th>
<th>Two-Sided Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>alternative Hypothesis</td>
<td>$H_A : p_1 - p_2 &lt; 0$</td>
</tr>
<tr>
<td>$p$-value</td>
<td>$P(Z &lt; z_{\text{obs}})$</td>
</tr>
</tbody>
</table>
Two p’s

\[ H_0 : p_1 = p_2 \quad \text{which is equivalent to} \quad H_0 : p_1 - p_2 = 0. \]

\[ Z_{obs} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p}) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \]

- Where the \( \hat{p} \) in the denominator is the combined (pooled) sample proportion.

= Total number of successes over total number of observations

So there are 3 different sample proportions: from sample one, from sample two and from both samples.

- P for P-value in a test hypothesis setting
- \( p \) for population proportion
- \( \hat{p} \) for sample proportion
- \( p_0 \) for the hypothesized population proportion value
Attendance Survey Question 24

- On a 4”x6” index card
  - Please write down your name and section number
  - Today’s Question:

  - Probability of making a type II error is denoted by:
    a. Alpha       b. Beta       c. Power

Example: compare 2 proportions

- A nation wide study: an aspirin every other day can sharply reduce a man’s risk of heart attack. (New York Times, reporting Jan. 27, 1987)

- Aspirin group: 104 Heart Att. in 11037
- Placebo group: 189 Heart Att. in 11034
- Randomized, double-blinded study

Example – cont.

- Let aspirin = group 1; placebo = group 2
  - \( p_1 \) = popu. proportion of Heart att. for group 1
  - \( p_2 \) = popu. proportion of Heart att. for group 2

\[ H_0: p_1 = p_2 \quad \text{which is equivalent to} \quad H_0: p_1 - p_2 = 0 \]

\[ H_a: p_1 \neq p_2 \quad \text{or} \quad H_a: p_1 - p_2 \neq 0 \]
Example – cont.

• We may use software to compute a p-value
• p-value = 7.71e-07 = 0.000000771

Or we can calculate by hand:

\[ z_{obs} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}} \]

Example – cont.

• n1 = 11037, n2 = 11034

\[ \hat{p}_1 = \frac{104}{11037} = 0.0094285 \]
\[ \hat{p}_2 = \frac{189}{11034} = 0.01712887 \]
\[ \hat{p} = \frac{104 + 189}{11037 + 11034} = 0.013275 \]
\[ z = \frac{-0.00770602}{0.001540777} = -5.001386 \]

Example – cont.

\[ z_{obs} = \frac{0.00942285 - 0.01712887}{\sqrt{\frac{0.00942285(1-0.00942285)}{11037} + \frac{0.013275(1-0.013275)}{11034}}} \]
Example – cont.

• P-value = 2 x P(Z > |-5.00|)
• It falls out of the range of our Z-table, so…….
  P-value is approx. zero. (much smaller than 0.0000)
What is alpha level? Say it was 0.01. Since P-value is smaller than alpha, we reject the null hypothesis.