Chapter 6: Practice/review problems

The collection of problems listed below contains questions taken from previous MA123 exams.

Extreme values problems on a closed interval

[1]. Suppose \( f(t) = \begin{cases} \sqrt{4-t} & \text{if } t < 4 \\ \sqrt{t-4} & \text{if } t \geq 4 \end{cases} \).

Find the minimum of \( f(t) \) on the interval \([0, 6]\).

(a) 0  (b) 2  (c) 4  (d) 6  (e) 8

[2]. Let \( g(s) = \frac{s-1}{s+1} \). Find the maximum of \( g(s) \) on the interval \([0, 2]\).

(a) \(-1/3\)  (b) 0  (c) \(1/3\)  (d) \(2/3\)  (e) Neither the maximum nor the minimum exists on the given interval.

[3]. Suppose \( f(t) = \begin{cases} t^2 - 2t + 2 & \text{if } t < 1 \\ t^3 & \text{if } t \geq 1 \end{cases} \).

Find the minimum of \( f(t) \) on the interval \([0, 2]\).

(a) -1  (b) 0  (c) 1  (d) 2  (e) 8

[4]. Let \( f(x) = 3x^2 + 6x + 4 \). Find the maximum value of \( f(x) \) on the interval \([-2, 1]\).

(a) 5  (b) 7  (c) 9  (d) 13  (e) -1

[5]. Let \( G(x) = \begin{cases} (x-3) + 6 & \text{if } x \geq 3 \\ -(x-3) + 6 & \text{if } x < 3 \end{cases} \).

Find the minimum of \( G(x) \) on the interval \([-10, 10]\).

(a) 3  (b) 1  (c) -6  (d) 19  (e) 6

[6]. Let \( g(s) = \frac{1}{s+1} \). Find the maximum of \( g(s) \) on the interval \([0, 2]\).

(a) -1  (b) 0  (c) 1  (d) 2  (e) Neither the maximum nor the minimum exists on the given interval.

[7]. Find the minimum value of \( f(x) = x^3 - 3x + 3 \) on the interval \([-2, 4]\).

(a) 2  (b) 1  (c) 0  (d) -1  (e) -2
[8]. Find the maximum of $g(t) = |t + 4| + 10$ on the interval $[-12, 12]$.

(a) 19  (b) 20  (c) 24  (d) 26  (e) 28

[9]. Find the minimum value of $f(x) = \sqrt{x^2 - 2x + 16}$ on the interval $[0, 5]$.

(a) 1  (b) 2  (c) $\sqrt{15}$  (d) $\sqrt{12}$  (e) 0

[10]. Let $f(x) = |x^2 - 1| + 2$. Find the minimum of $f(x)$ on the interval $[-3, 3]$.

(a) 3  (b) 0  (c) 1  (d) 2  (e) −1

[11]. Suppose $f(t) = 2t^3 - 9t^2 + 12t + 31$. Find the value of $t$ in the interval $[0, 3]$ where $f(t)$ takes on its minimum.

(a) 0  (b) 1  (c) 2  (d) 3  (e) Neither the maximum nor the minimum exists on the given interval.

[12]. Let $Q(t) = t^2$. Find a value $A$ such that the average rate of change of $Q(t)$ from 1 to $A$ equals the instantaneous rate of change of $Q(t)$ at $t = 2A$

(a) 1  (b) $\frac{1}{3}$  (c) $\frac{1}{4}$  (d) $\frac{1}{5}$  (e) Does not exist

[Mean Value Theorem problems]

[13]. Find the value of $A$ such that the average rate of change of the function $g(s) = s^3$ on the interval $[0, A]$ is equal to the instantaneous rate of change of the function at $s = 1$.

(a) $\sqrt{2}$  (b) $\sqrt{3}$  (c) $\sqrt{5}$  (d) $\sqrt{6}$  (e) $\sqrt{12}$

[14]. Suppose $k(s) = s^2 + 3s + 1$. Find a value $c$ in the interval $[1, 3]$ such that $k'(c)$ equals the average rate of change of $k(s)$ on the interval $[1, 3]$.

(a) −1  (b) 0  (c) 1  (d) 2  (e) 3

[15]. Let $k(x) = x^3 + 2x$. Find a value of $c$ between 1 and 3 such that the average rate of change of $k(x)$ from $x = 1$ to $x = 3$ is equal to the instantaneous rate of change of $k(x)$ at $x = c$.

(a) 30  (b) 15  (c) $\sqrt{\frac{28}{3}}$  (d) $\sqrt{\frac{13}{3}}$  (e) 60

[Increasing/decreasing problems]

[16]. Which function is always increasing on $(0, 2)$

(a) $\sqrt{x} + x^2$  (b) $x + \frac{1}{x}$  (c) $x^3 - 3x$

(d) $7 - |x|$  (e) $(x - 1)^4$
17. Suppose that a function \( f(x) \) has derivative \( f'(x) = x^2 + 1 \). Which of the following statements is true about the graph of \( y = f(x) \)?

(a) The function is increasing on \((-\infty, \infty)\)
(b) The function is decreasing on \((-\infty, \infty)\)
(c) The function is increasing on \((-\infty, -1)\) and \((1, \infty)\), and the function is decreasing on \((-1, 1)\).
(d) The function is increasing on \((-\infty, 0)\), and the function is decreasing on \((0, \infty)\).
(e) The function is decreasing on \((-\infty, 0)\), and the function is increasing on \((0, \infty)\).

18. Find the largest value of \( A \) such that the function \( g(s) = s^3 - 3s^2 - 24s + 1 \) is increasing on the interval \((-5, A)\).

(a) -4  (b) -2  (c) 0  (d) 2  (e) 4

19. Let \( f(x) = e^{-x^2} \). Find the intervals where \( f(x) \) is decreasing.

(a) \((-\infty, 0)\)  (b) \((0, \infty)\)  (c) \((-\infty, -1)\)
(d) \((1, \infty)\)  (e) \((-1, 1)\)

20. Let \( f(x) = x \ln x \). Find the intervals where \( f(x) \) is increasing.

(a) \((0, \infty)\)  (b) \((1, \infty)\)  (c) \((e, \infty)\)
(d) \((1/e, \infty)\)  (e) \((1/e, e)\)

21. Suppose the cost, \( C(q) \), of stocking a quantity \( q \) of a product equals \( C(q) = \frac{100}{q} + q \). The rate of change of the cost with respect to \( q \) is called the marginal cost. When is the marginal cost positive?

(a) \( q > 10 \)  (b) \( q > 15 \)  (c) \( q < 20 \)  (d) \( q < 25 \)  (e) \( q = 30 \)

22. For which values of \( t \) is the function \( t^3 - 2t + 1 \) increasing?

(a) \( t > \sqrt{2/3} \) or \( t < -\sqrt{2/3} \)  (b) \( -\sqrt{2/3} < t < \sqrt{2/3} \)  (c) \( 0 < t < \sqrt{4/3} \)
(d) \( -\sqrt{4/3} < t < 0 \)  (e) Never

23. Suppose that \( g'(x) = x^2 - x - 6 \). Find the interval(s) where \( g(x) \) is increasing.

(a) \((-1, 2)\)  (b) \((-\infty, -2) \) and \((3, \infty)\)  (c) \((-\infty, -1)\) and \((2, \infty)\)
(d) \((-2, 3)\)  (e) It cannot be determined from the information given

24. Let \( f(x) = xe^{2x} \). Then \( f \) is decreasing on the following interval.

(a) \((-\infty, -1/2)\)  (b) \((-1/2, \infty)\)  (c) \((-\infty, 1/2)\)
(d) \((1/2, \infty)\)  (e) \((-\infty, 0)\)
[25]. Find the interval(s) where \( f(x) = -x^3 + 18x^2 - 105x + 4 \) is increasing.
   (Note that the coefficient of \( x^3 \) is \(-1\), so compute carefully.)
   \( f'(x) = -3x^2 + 36x - 105 \)
   (a) \((-\infty, 5)\) and \((7, \infty)\)  
   (b) \((5, 7)\)  
   (c) \((-\infty, -5)\) and \((7, \infty)\)  
   (d) \((-5, 7)\)  
   (e) \((-7, 5)\)

[26]. Suppose that \( f(x) = xg(x) \), and for all positive values of \( x \) the function \( g(x) \) is negative (i.e., \( g(x) < 0 \)) and decreasing. Which of the following is true for the function \( f(x) \)?
   (a) \( f(x) \) is negative and decreasing for all positive values of \( x \).  
   (b) \( f(x) \) is positive and increasing for all positive values of \( x \).  
   (c) \( f(x) \) is negative and increasing for all positive values of \( x \).  
   (d) \( f(x) \) is positive and decreasing for all positive values of \( x \).  
   (e) None of the above.

[27]. Suppose the derivative of a function \( g(x) \) is given by \( g'(x) = x^2 - 1 \). Find all intervals on which \( g(x) \) is increasing.
   (a) \((-\infty, \infty)\)  
   (b) \((-1, 1)\)  
   (c) \((-\infty, -1)\) and \((1, \infty)\)  
   (d) \((0, \infty)\)  
   (e) \((-\infty, 0)\)

**Extreme values problems using the first derivative**

[28]. Suppose the derivative of the function \( h(x) \) is given by \( h'(x) = 1 - |x| \). Find the value of \( x \) in the interval \([-1, 1]\) where \( h(x) \) takes on its minimum value.
   (a) \(-1/2\)  
   (b) \(-1\)  
   (c) \(0\)  
   (d) \(1/2\)  
   (e) \(1\)

[29]. Suppose the total cost, \( C(q) \), of producing a quantity \( q \) of a product equals
   \[ C(q) = 1000 + q + \frac{1}{10}q^2. \]
   The average cost, \( A(q) \), equals the total cost divided by the quantity produced. What is the minimum average cost? (Assume \( q > 0 \))
   (a) \(20\)  
   (b) \(21\)  
   (c) \(26\)  
   (d) \(30\)  
   (e) \(31\)

[30]. Suppose that a function \( h(x) \) has derivative \( h'(x) = x^2 + 4 \). Find the \( x \) value in the interval \([-1, 3]\) where \( h(x) \) takes its minimum.
   (a) \(-1\)  
   (b) \(3\)  
   (c) \(5\)  
   (d) \(13\)  
   (e) \(29\)

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[31]. Suppose the cost, $C(q)$, of stocking a quantity $q$ of a product equals $C(q) = \frac{100}{q} + q$. Which positive value of $q$ gives the minimum cost?

(a) 10  (b) 15  (c) 20  (d) 25  (e) 30

[32]. Find a local extreme point of $f(x) = \frac{\ln x}{x}$.

(a) $(1,0)$ is a local maximum point.  (b) $(1,0)$ is a local minimum point.
  (c) $(e,1/e)$ is a local minimum point.  (d) $(e,1/e)$ is a local maximum point.
  (e) $f(x)$ has no local extreme points.

[33]. Suppose the derivative of $G(q)$ is given by $G'(q) = q^2(q + 1)^2(q + 2)^2$. Find the value of $q$ in the interval $[-5,5]$ where $G(q)$ takes on its maximum.

(a) $-5$  (b) $-2$  (c) $-1$  (d) 0  (e) 5

[34]. Suppose the derivative of $H(s)$ is given by $H'(s) = s^2(s + 1)$. Find the value of $s$ in the interval $[-100,100]$ where $H(s)$ takes on its minimum.

(a) $-100$  (b) $-1$  (c) 0  (d) 1  (e) 100

**Concavity problems**

[35]. Find the intervals where $f(x) = x^4 - 12x^3 + 48x^2 + 10x - 8$ is concave downward.

(a) $(-\infty, \infty)$  (b) $(1, \infty)$  (c) $(-\infty, -4)$ and $(-2, \infty)$
  (d) $(-\infty, 2)$ and $(4, \infty)$  (e) $(2, 4)$

[36]. Let $f(x) = e^{-x^2}$. Find the intervals where $f(x)$ is concave upward.

(a) $(1, \infty)$  (b) $(-e, e)$  (c) $(-\infty, -\sqrt{1/2})$ and $(\sqrt{1/2}, \infty)$
  (d) $(-\sqrt{1/2}, \sqrt{1/2})$  (e) $(-\infty, -e)$ and $(e, \infty)$

[37]. Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is concave downward.

(a) $(0,1)$  (b) $(0, \infty)$  (c) $(0, 1/e)$
  (d) $(1/e, \infty)$  (e) $f(x)$ is not concave downward anywhere

[38]. Suppose that the derivative of $f(x)$ is given by $f'(x) = x^2 - 5x + 6$. Then the graph of $f(x)$ is concave downward on the following intervals(s).

(a) $(-\infty, 2)$ and $(3, \infty)$  (b) $(2, 3)$  (c) $(-\infty, 2,5)$
  (d) $(2.5, \infty)$  (e) $f(x)$ in not concave downward on any interval

[39]. Find the interval(s) where the graph of $f(x) = x^4 + 18x^3 + 120x^2 + 10x + 50$ is concave downward.

(a) $(-5,4)$  (b) $(4,5)$  (c) $(-\infty, 4)$ and $(5, \infty)$
  (d) $(-5, -4)$  (e) $(-\infty, -5)$ and $(4, \infty)$