Chapter 6: Practice/review problems

The collection of problems listed below contains questions taken from previous MA123 exams.

Extreme values problems on a closed interval

[1]. Suppose \( f(t) = \begin{cases} \sqrt{4-t} & \text{if } t < 4 \\ \sqrt{t-4} & \text{if } t \geq 4 \end{cases} \).

Find the minimum of \( f(t) \) on the interval \([0,6]\).

(a) 0 (b) 2 (c) 4 (d) 6 (e) 8

[2]. Let \( g(s) = \frac{s-1}{s+1} \). Find the maximum of \( g(s) \) on the interval \([0,2]\).

(a) \(-1/3\) (b) 0 (c) \(1/3\) (d) \(2/3\) (e) Neither the maximum nor the minimum exists on the given interval.

[3]. Suppose \( f(t) = \begin{cases} t^2 - 2t + 2 & \text{if } t < 1 \\ t^3 & \text{if } t \geq 1 \end{cases} \).

Find the minimum of \( f(t) \) on the interval \([0,2]\).

(a) \(-1\) (b) 0 (c) 1 (d) 2 (e) 8

[4]. Let \( f(x) = 3x^2 + 6x + 4 \). Find the maximum value of \( f(x) \) on the interval \([-2,1]\).

(a) 5 (b) 7 (c) 9 (d) 13 (e) \(-1\)

[5]. Let \( G(x) = \begin{cases} (x-3) + 6 & \text{if } x \geq 3 \\ -(x-3) + 6 & \text{if } x < 3 \end{cases} \).

Find the minimum of \( G(x) \) on the interval \([-10,10]\).

(a) 3 (b) 1 (c) \(-6\) (d) 19 (e) 6

[6]. Let \( g(s) = \frac{1}{s+1} \). Find the maximum of \( g(s) \) on the interval \([0,2]\).

(a) \(-1\) (b) 0 (c) 1 (d) 2 (e) Neither the maximum nor the minimum exists on the given interval.

[7]. Find the minimum value of \( f(x) = x^3 - 3x + 3 \) on the interval \([-2,4]\).

(a) 2 (b) 1 (c) 0 (d) \(-1\) (e) \(-2\)
[8]. Find the maximum of \( g(t) = |t + 4| + 10 \) on the interval \([-12, 12]\).

(a) 19  (b) 20  (c) 24  (d) 26  (e) 28

[9]. Find the minimum value of \( f(x) = \sqrt{x^2 - 2x + 16} \) on the interval \([0, 5]\).

(a) 1  (b) 2  (c) \( \sqrt{15} \)  (d) \( \sqrt{12} \)  (e) 0

[10]. Let \( f(x) = |x^2 - 1| + 2 \). Find the minimum of \( f(x) \) on the interval \([-3, 3]\).

(a) 3  (b) 0  (c) 1  (d) 2  (e) -1

[11]. Suppose \( f(t) = 2t^3 - 9t^2 + 12t + 31 \). Find the value of \( t \) in the interval \([0, 3]\) where \( f(t) \) takes on its minimum.

(a) 0  (b) 1  (c) 2  (d) 3  (e) Neither the maximum nor the minimum exists on the given interval.

[12]. Let \( Q(t) = t^2 \). Find a value \( A \) such that the average rate of change of \( Q(t) \) from 1 to \( A \) equals the instantaneous rate of change of \( Q(t) \) at \( t = 2A \)

(a) 1  (b) \( \frac{1}{3} \)  (c) \( \frac{1}{4} \)  (d) \( \frac{1}{5} \)  (e) Does not exist

Mean Value Theorem problems

[13]. Find the value of \( A \) such that the average rate of change of the function \( g(s) = s^3 \) on the interval \([0, A]\) is equal to the instantaneous rate of change of the function at \( s = 1 \).

(a) \( \sqrt{2} \)  (b) \( \sqrt{3} \)  (c) \( \sqrt{5} \)  (d) \( \sqrt{6} \)  (e) \( \sqrt{12} \)

[14]. Suppose \( k(s) = s^2 + 3s + 1 \). Find a value \( c \) in the interval \([1, 3]\) such that \( k'(c) \) equals the average rate of change of \( k(s) \) on the interval \([1, 3]\).

(a) -1  (b) 0  (c) 1  (d) 2  (e) 3

[15]. Let \( k(x) = x^3 + 2x \). Find a value of \( c \) between 1 and 3 such that the average rate of change of \( k(x) \) from \( x = 1 \) to \( x = 3 \) is equal to the instantaneous rate of change of \( k(x) \) at \( x = c \).

(a) 30  (b) 15  (c) \( \sqrt{\frac{28}{3}} \)  (d) \( \sqrt{\frac{13}{3}} \)  (e) 60

Increasing/decreasing problems

[16]. Which function is always increasing on \((0, 2)\)

(a) \( \sqrt{x} + x^2 \)  (b) \( x + (1/x) \)  (c) \( x^3 - 3x \)

(d) \( 7 - |x| \)  (e) \((x-1)^4\)
[17]. Suppose that a function \( f(x) \) has derivative \( f'(x) = x^2 + 1 \). Which of the following statements is true about the graph of \( y = f(x) \)?

(a) The function is increasing on \((-\infty, \infty)\)
(b) The function is decreasing on \((-\infty, \infty)\)
(c) The function is increasing on \((-\infty, -1)\) and \((1, \infty)\), and the function is decreasing on \((-1, 1)\).
(d) The function is increasing on \((-\infty, 0)\), and the function is decreasing on \((0, \infty)\).
(e) The function is decreasing on \((-\infty, 0)\), and the function is increasing on \((0, \infty)\).

[18]. Find the largest value of \( A \) such that the function \( g(s) = s^3 - 3s^2 - 24s + 1 \) is increasing on the interval \((-5, A)\).

(a) -4  (b) -2  (c) 0  (d) 2  (e) 4

[19]. Let \( f(x) = e^{-x^2} \). Find the intervals where \( f(x) \) is decreasing.

(a) \((-\infty, 0)\)  (b) \((0, \infty)\)  (c) \((-\infty, -1)\)
(d) \((1, \infty)\)  (e) \((-1, 1)\)

[20]. Let \( f(x) = x \ln x \). Find the intervals where \( f(x) \) is increasing.

(a) \((0, \infty)\)  (b) \((1, \infty)\)  (c) \((e, \infty)\)
(d) \((1/e, \infty)\)  (e) \((1/e, e)\)

[21]. Suppose the cost, \( C(q) \), of stocking a quantity \( q \) of a product equals \( C(q) = \frac{100}{q} + q \). The rate of change of the cost with respect to \( q \) is called the marginal cost. When is the marginal cost positive?

(a) \( q > 10 \)  (b) \( q > 15 \)  (c) \( q < 20 \)  (d) \( q < 25 \)  (e) \( q = 30 \)

[22]. For which values of \( t \) is the function \( t^3 - 2t + 1 \) increasing?

(a) \( t > \sqrt{2/3} \) or \( t < -\sqrt{2/3} \)  (b) \( -\sqrt{2/3} < t < \sqrt{2/3} \)  (c) \( 0 < t < \sqrt{4/3} \)
(d) \( -\sqrt{4/3} < t < 0 \)  (e) Never

[23]. Suppose that \( g'(x) = x^2 - x - 6 \). Find the interval(s) where \( g(x) \) is increasing.

(a) \((-1, 2)\)  (b) \((-\infty, -2)\) and \((3, \infty)\)  (c) \((-\infty, -1)\) and \((2, \infty)\)
(d) \((-2, 3)\)  (e) It cannot be determined from the information given

[24]. Let \( f(x) = xe^{2x} \). Then \( f \) is decreasing on the following interval.

(a) \((-\infty, -1/2)\)  (b) \((-1/2, \infty)\)  (c) \((-\infty, 1/2)\)
(d) \((1/2, \infty)\)  (e) \((-\infty, 0)\)
[25]. Find the interval(s) where \( f(x) = -x^3 + 18x^2 - 105x + 4 \) is increasing. (Note that the coefficient of \( x^3 \) is -1, so compute carefully.)

(a) \((-\infty, 5)\) and \((7, \infty)\)  
(b) \((5, 7)\)  
(c) \((-\infty, -5)\) and \((7, \infty)\)  
(d) \((-5, 7)\)  
(e) \((-7, 5)\)

[26]. Suppose that \( f(x) = xg(x) \), and for all positive values of \( x \) the function \( g(x) \) is negative (i.e., \( g(x) < 0 \)) and decreasing. Which of the following is true for the function \( f(x) \)?

(a) \( f(x) \) is negative and decreasing for all positive values of \( x \).  
(b) \( f(x) \) is positive and increasing for all positive values of \( x \).  
(c) \( f(x) \) is negative and increasing for all positive values of \( x \).  
(d) \( f(x) \) is positive and decreasing for all positive values of \( x \).  
(e) None of the above.

[27]. Suppose the derivative of a function \( g(x) \) is given by \( g'(x) = x^2 - 1 \). Find all intervals on which \( g(x) \) is increasing.

(a) \((-\infty, \infty)\)  
(b) \((-1, 1)\)  
(c) \((-\infty, -1)\) and \((1, \infty)\)  
(d) \((0, \infty)\)  
(e) \((-\infty, 0)\)

**Extreme values problems using the first derivative**

[28]. Suppose the derivative of the function \( h(x) \) is given by \( h'(x) = 1 - |x| \). Find the value of \( x \) in the interval \([-1, 1]\) where \( h(x) \) takes on its minimum value.

(a) \(-1/2\)  
(b) \(-1\)  
(c) \(0\)  
(d) \(1/2\)  
(e) \(1\)

[29]. Suppose the total cost, \( C(q) \), of producing a quantity \( q \) of a product equals

\[
C(q) = 1000 + q + \frac{1}{10}q^2.
\]

The average cost, \( A(q) \), equals the total cost divided by the quantity produced. What is the minimum average cost? (Assume \( q > 0 \))

(a) 20  
(b) 21  
(c) 26  
(d) 30  
(e) 31

[30]. Suppose that a function \( h(x) \) has derivative \( h'(x) = x^2 + 4 \). Find the \( x \) value in the interval \([-1, 3]\) where \( h(x) \) takes its minimum.

(a) \(-1\)  
(b) 3  
(c) 5  
(d) 13  
(e) 29
[31]. Suppose the cost, \( C(q) \), of stocking a quantity \( q \) of a product equals \( C(q) = \frac{100}{q} + q \). Which positive value of \( q \) gives the minimum cost?

(a) 10  
(b) 15  
(c) 20  
(d) 25  
(e) 30

[32]. Find a local extreme point of \( f(x) = \frac{\ln x}{x} \).

(a) \((1,0)\) is a local maximum point.  
(b) \((1,0)\) is a local minimum point.  
(c) \((e,1/e)\) is a local minimum point.  
(d) \((e,1/e)\) is a local maximum point.  
(e) \(f(x)\) has no local extreme points.

[33]. Suppose the derivative of \( G(q) \) is given by \( G'(q) = q^2(q + 1)^2(q + 2)^2 \). Find the value of \( q \) in the interval \([-5, 5]\) where \( G(q) \) takes on its maximum.

(a) -5  
(b) -2  
(c) -1  
(d) 0  
(e) 5

[34]. Suppose the derivative of \( H(s) \) is given by \( H'(s) = s^2(s+1) \). Find the value of \( s \) in the interval \([-100, 100]\) where \( H(s) \) takes on its minimum.

(a) -100  
(b) -1  
(c) 0  
(d) 1  
(e) 100

[35]. Find the intervals where \( f(x) = x^4 - 12x^3 + 48x^2 + 10x - 8 \) is concave downward.

(a) \((-\infty, \infty)\)  
(b) \((1, \infty)\)  
(c) \((-\infty, -4)\) and \((-2, \infty)\)  
(d) \((-\infty, 2)\) and \((4, \infty)\)  
(e) \((2, 4)\)

[36]. Let \( f(x) = e^{-x^2} \). Find the intervals where \( f(x) \) is concave upward.

(a) \((1, \infty)\)  
(b) \((-\infty, e)\)  
(c) \((-\infty, -\sqrt{1/2})\) and \((\sqrt{1/2}, \infty)\)  
(d) \((-\sqrt{1/2}, \sqrt{1/2})\)  
(e) \((-\infty, -e)\) and \((e, \infty)\)

[37]. Let \( f(x) = x \ln x \). Find the intervals where \( f(x) \) is concave downward.

(a) \((0,1)\)  
(b) \((0, \infty)\)  
(c) \((0, 1/e)\)  
(d) \((1/e, \infty)\)  
(e) \(f(x)\) is not concave downward anywhere

[38]. Suppose that the derivative of \( f(x) \) is given by \( f'(x) = x^2 - 5x + 6 \). Then the graph of \( f(x) \) is concave downward on the following intervals(s).

(a) \((-\infty, 2)\) and \((3, \infty)\)  
(b) \((2, 3)\)  
(c) \((-\infty, 2.5)\)  
(d) \((2.5, \infty)\)  
(e) \(f(x)\) in not concave downward on any interval

[39]. Find the interval(s) where the graph of \( f(x) = x^4 + 18x^3 + 120x^2 + 10x + 50 \) is concave downward.

(a) \((-5, 4)\)  
(b) \((4, 5)\)  
(c) \((-\infty, 4)\) and \((5, \infty)\)  
(d) \((-5, -4)\)  
(e) \((-\infty, -5)\) and \((-4, \infty)\)