4 The Cartesian Coordinate Practice Problems

1. Is \((3, 2)\) on the graph of \(x^2 - y^3 = 1\)? Yes.
2. Is \((0, 1)\) on the graph of \(x^2 - y^3 = 1\)? No.
3. Is \((0, -1)\) on the graph of \(x^2 - y^3 = 1\)? Yes.
4. Find the intercepts of the graph of \(x^2 - y^3 = 1\).
   \(x\)-intercepts: \((-1, 0)\) and \((1, 0)\); \(y\)-intercept: \((0, -1)\)
5. Find the point on the \(x\)-axis that is equidistant to \((2, 5)\) and \((-1, 3)\).
   \((\frac{19}{6}, 0)\)
6. Find the point on the \(y\)-axis that is equidistant to \((2, 5)\) and \((-1, 3)\).
   \((0, \frac{19}{4})\)
7. Find the area of the triangle with vertices \(A(-2, -5), B(-2, 7),\) and \(C(10, 10)\). 72
8. Show that the triangle whose vertices are \(A(4, 15), B(12, 7),\) and \(C(-1, 2)\) is isosceles.
   \(\overline{AC} = \overline{BC} = \sqrt{194}\)
9. Sketch the graph of the circle defined by \((x + 5)^2 + y^2 = 16\). What are the center and radius of this circle? Center: \((-5, 0)\); Radius: 4
10. Is the graph of \(x^2 + 6x + y^2 - 10y + 26 = 0\) a circle? If so, find its center and radius.
    Yes, the equation is equivalent to \((x + 3)^2 + (y - 5)^2 = 8\).
    Center: \((-3, 5)\); Radius: \(\sqrt{8}\).
11. Is the graph of \(4x^2 - 8x + 4y^2 + 4y - 23 = 0\) a circle? If so, find its center and radius.
    Yes, the equation is equivalent to \((x - 1)^2 + (y + \frac{1}{2})^2 = 7\).
    Center: \((1, -\frac{1}{2})\); Radius: \(\sqrt{7}\).
12. Is the graph of \(x^2 - 2x + y^2 + 8y + 26 = 0\) a circle? If so, find its center and radius.
    No, the equation is equivalent to \((x - 1)^2 + (y + 4)^2 = -9\), which is not an equation of a circle.
13. Describe the graph of \(x^2 + 4x + y^2 + 10y + 29 = 0\).
    The equation is equivalent to the standard equation of a circle with center \((-2, -5)\) and radius 0. Thus, the graph is the single point \((-2, -5)\).
14. A diameter of a circle has endpoints \((1, -2)\) and \((3, 6)\). Find an equation for the circle.
    \((x - 2)^2 + (y - 2)^2 = 17\)
15. The center of a circle is \((5, -2)\), and circle passes through the point \((-2, 3)\). Find an equation for the circle.
\[(x - 5)^2 + (y + 2)^2 = 74\]

16. **TRUE or FALSE:** The line through the points \((0, -1)\) and \((-1, 4)\) is perpendicular to the line through the points \((2, -8)\) and \((7, -7)\). **True**

17. **TRUE or FALSE:** The line through the points \((-5, -7)\) and \((-8, -5)\) is parallel to the line through the points \((-7, 0)\) and \((-10, 2)\). **True**

18. Find the intercept(s) of the graph of \((x - 1)^2 + (y + 5)^2 = 17\).

   **No x-intercepts. Two y-intercepts:** \((0, -1)\) and \((0, -9)\).

19. The center of a circle is \((4, -5)\) and the circle intersects the \(x\)-axis at 2 and 6. Find an equation for the circle.
\[(x - 4)^2 + (y + 5)^2 = 29\]

20. For each point, determine if the point is inside, outside, or on the circle
\[(x + 5)^2 + (y - 3)^2 = 36\]

   (a) \((4, 2)\) **outside** the circle
   (b) \((-5, 0)\) **inside** the circle
   (c) \((1, 2)\) **outside** the circle

21. Which of the following are equations for the line through the points \(P(1, 5)\) and \(Q(2, -3)\)? **The answers are in bold.**

   (a) \(y + 3 = -8(x - 2)\)
   (b) \(y = -8x - 4\)
   (c) \(y = -8(x - 1) + 5\)
   (d) \(y + 3 = \frac{-1}{8}(x - 2)\)
   (e) \(y + 3 = \frac{1}{8}(x - 2)\)
   (f) \(y - 5 = \frac{-1}{8}(x - 1)\)
   (g) \(y - 5 = \frac{1}{8}(x - 1)\)
   (h) \(y - 5 = -8(x - 1)\)
   (i) \(y + 5 = -8(x + 1)\)
   (j) \(y - 5 = -8x - 1\)
   (k) \(y - 5 = \frac{-1}{8}x - 1\)
22. Find an equation for the line that is parallel to \( y = \frac{5}{6}x + 4 \) and passes through the point \((0,12)\).
\[
y = \frac{5}{6}x + 12
\]

23. Find an equation for the line that is parallel to \( y = \frac{5}{6}x + 7 \) and contains the point \((3,21)\).
\[
y - 21 = \frac{5}{6}(x - 3)
\]

24. Find an equation for the line that is perpendicular to \( y = \frac{5}{6}x + 4 \) and contains the point \((0,14)\).
\[
y = -\frac{6}{5}x + 14
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